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A Theory for Parsimonious Voice-Leading Classes

Uma teoria para classes de condução parcimoniosa de vozes

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Abstract: This article introduces a theoretical-analytical model intended to address voice leading in popular music, considering specifically harmonic progressions characterized by dense chordal formations and smooth voicing connections, which is perfectly epitomized in Antonio Carlos Jobim's compositions. New concepts, typologies, preference rules, graphic representations, specific terminology/symbology, as well as an integrated analytical model provide the necessary foundation for the elaboration of a system of classes of parsimonious voice leading (PVL), which form the very core of the proposal. Some analytical application is provided in the last section of the article with the exam of five excerpts of Jobim's compositions.

Keywords: Voice leading. Jazz and bossa music. Antônio Carlos Jobim's harmony. Parsimonious intervals. Parsimonious voice-leading classes.

Resumo: Este artigo apresenta um modelo teórico-analítico destinado ao exame de conduções de vozes em música popular, considerando especificamente progressões harmônicas caracterizadas por formações acordais densas e conexões melódicas econômicas, o que é perfeitamente epitomizado pelas composições de Antônio Carlos Jobim. Novos conceitos, tipologias, regras de preferência, terminologias e simbologias específicas, assim como um modelo analítico integrado fornece a fundamentação necessária para a elaboração de um sistema de classes de conduções de vozes parcimoniosas, o que forma o núcleo desta proposta. Aplicações analíticas são apresentadas na seção final do artigo, com o exame de excertos de composições de Jobim.

Palavras-chave: Condução de vozes. Jazz e bossa nova. Harmonia de Antônio Carlos Jobim. Intervalos parcimoniosos. Classes de condução de vozes parcimoniosas.

This article introduces an original theory centered on the melodic relations between chord notes in harmonic progressions used in a specific subset of popular music. Such particular universe encompasses those genres characterized by a strong presence of medium-to-high chordal density (i.e., four to seven distinct pitch classes per chord), resulting from inclusion in the basic triadic structures of sixths, sevenths, as well as harmonic extensions, like ninths, elevenths, and thirteenths, especially those chromatically altered. By an initial hypothesis, these extensions are used in such contexts not only for enriching and/or making denser the chords, but also for optimizing economic voice leading in chordal progressions. The music of Brazilian composer Antônio Carlos Jobim (1927-1994) epitomizes the repertoire aimed in the present theory, and was accordingly selected for application of the analytical methodology that is developed along the study. Although Jobim is normally associated with the aesthetic of bossa nova (or simply bossa), arisen in Rio de Janeiro city at the end of 1950s, his style comprises a quite diversified palette of distinct colors, distributed into a number of compositional phases. More importantly than aesthetic issues for the present goals, however, is the fact that Jobim's harmony (apart from stylistic particularities) is especially characterized by the use of a wide spectrum of chordal qualities,[1] smoothly connected by highly chromatic voice leading. As a matter of fact, harmony is paramount for the understanding of Jobim's peculiar style. In a book dedicated to the investigation of the structure of Jobim's music, Peter Freeman attributes his

> idiosyncratic approach to harmony (as distinct from melody, rhythm or lyrical content) is a compelling and profound characteristic of his musical style. His harmonic progressions consist for the most part of complex (altered and extended) chords, often chosen to highlight downward chromatic relationships between successive chords built as support to simple sequential melodic motives" (Freeman 2019, p. 100).

Although these elements integrate Jobim's harmony in intensities probably unmatched, they are certainly not exclusive to his music. Essentially, such attributes describe in a broad manner the universe covered by the present proposal. The first part of this study is dedicated to the formalization of a comprehensive theory on voice-leading relations in musical contexts that share

[1] As evidenced by a recent analysis of his complete song collection (author in preparation).

the properties listed by Freeman. As far as one knows, a number of specific questions directly associate with the particularities of such repertoire were not yet properly addressed, which motivated the formulation of new concepts, classifications, and principles that define the present theoretical framework. These principles are organized into three basic axes, whose scopes concern (1) a systematic approach in respect of connection of voices in contiguous chords; (2) a typology for abstracted smooth melodic linkages; (c) the contextualization of these relations, taking into account the harmonic functions of the individual notes. The discussion includes the elaboration of an analytical methodology to be applied in the second part of the article (section 5), addressing selected passages from Jobim's repertoire.

1. Background

Recently, the systematical study of voice leading became one of the most attracting branches of transformational and neo-Riemannian theories.[2] In the late 1990s, Richard Cohn popularized the concept of "parsimony",[3] which since then came to be employed in related neo-Riemannian contexts as a quality of voice leading, chords, and even *Tonnetze*. Cohn associates basically the melodic version of the principle with the "law of shortest way",[4] which implicitly also links parsimony to the physical tendency for lesser effort and to the economy of means. In Cohn's original formulation, parsimonious motions are defined as those that occur between voices of two perfect, mode-opposed triads involved in the PLR family of operations (Parallel / *Leittonwechsel* / Relative), considering, therefore, that two notes of the first chord are maintained during the transformation and the third one moves by a minor second (operations P and L) or a major second (in the case of operation R). Jack Douthett and Peter Steinbach 1998, followed by Guy Capuzzo 2004, considered more flexible definitions for parsimony, stating that it is sufficient that just one note is maintained during the

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2 For some distinct approaches, see Callender (1998), Douthett; Steinbach (1998), Capuzzo (2004), Cohn (1997; 1998; 2012), Callender; Quinn; Tymoczko (2008), and Tymoczko (2011,; 2018).

3 As stated by Cohn himself (1997, p. 62), the term was introduced by Czech theorist Ottokar *Hostinský (*a follower of Riemann's theory) *in a book published in 1879, Die Lehre von den musikalischen Klängen*.

4 Whose authorship is sometimes attributed either to Arnold Schoenberg or Siegfried Dehn.

voice leading, with the other two moving by minor or major second. The present proposal adopts a still broader meaning for the term, directing the focus from the whole chordal voicing to the individual melodic behavior.[5] In this conception, a parsimonious motion between two pitches is equal to or lesser than a (descending or ascending) major second, including the possibility of absence of real motion (or, more formally, a zero-semitone connection).

In his 2012's book "Audacious Euphony", Cohn refined his previous voicing-leading theoretical discussion, introducing the ideas of *unit of voice-leading work* ("the motion of a voice by one semitone"), and *idealized voice leading* (Cohn 2012, p. 6). The latter concept addresses the possibility of reconfiguring a musical surface (mainly with analytical purposes) in a manner that the smooth connections and common-tone relations between the chordal voices become more evident.[6] In idealized voice-leading reductions, octave displacements, doublings, and apparent voice crossings are eliminated, depicting abstract constructions of "purer" smooth voice leadings.

Dmitri Tymoczko's celebrated book "A Geometry of Music" (2011) has plenty of discussions about the melodic connections between several types of *musical objects* (scales, chords, pitch collections, or, in his terms, *macro-harmonies*), in which the principle of efficient voice leading occupies a central position. For Tymoczko, part of the harmony of Romantic chromatic repertoire and of jazz music can be understood either as predominantly resulting from melodic strategies or, at least, a combination of vertical and horizontal conceptions.[7] As he affirms, "harmony and counterpoint constrain one another… Clearly, efficient voice leading is simply conjunct melodic motion in all parts of a contrapuntal texture." (Tymoczko 2011, p. 12–13).[8] Tymoczko also addresses pitch-proximity

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[5] This point has as a cognitive support, the gestalt principle of proximity, which favors the perception of a sequence of notes in step motion as forming a unique stream (Lipscomb 1996, p. 147–50). The proximity principle is also a decisive factor in melodic streaming for David Huron 2016.

[6] See also Capuzzo 2004 for the related concepts of *p parsimony and pc parsimony*.

[7] See especially the geometric three- and four-dimensional projections of these relations proposed by the author as a precise means to depict them (Tymoczko 2011, p. 85–115).

[8] In a recent article, Tymoczko 2018 presents the idea of *iterable voice leading,* proposing a formal approach able to be applied in analyses of voice-leading patterns. In a sense, the present study

calculation, considering three alternatives (p. 398–400): (i) largest-distance metric (in which voice leadings are compared by their largest element); (ii) "taxicab" metric (in which the total distance, in semitones, of the component motions of a voice leading is counted); and (iii) Euclidean metric (in which intervallic distances are measured by their geometric projection in his Möbius-strip voice-leading scheme). The measurement of distances is a central aspect of his concept of *voice-leading efficiency*.

An important work focusing the subject on a cognitive point of view is the recent book "Voice Leading: The Science Behind a Musical Art", written by David Huron in 2016. Huron makes a very comprehensive and in-depth investigation about the elements and factors involved in the perception of the contrapuntal relations in polyphonic music (which also involves the point of view of a composer), through a system of principles and preference rules, a perspective that considerably influenced the formalization of the present proposal.

A number of studies address the issue of voice leading involved in connections between pitch-class sets in post-tonal music. David Lewin's pioneering in this field is attested by his transformation called IFUNC(X,Y)(i), a function that "tells us in how many different ways the interval i can be spanned between (members of) [set classes] X and (members of) Y." (Lewin 1987, p. 88). Robert Morris 1998 introduced the ideas of *voice-leading spaces* and *total voice leading*. The latter concept corresponds to the set of all possible motions between the pitch classes of two contiguous chords. This can be correlated to conventional, tonal voice leading, where pitches are individually mapped onto their sequential continuations in the respective lines. This view is adopted by Joseph Straus 2014, who reformulates Morris's principle and associates it with Lewin's IFUNC(X,Y), creating an IFUNC-vector, intended to systematically explore connections between members of contiguous pitch-class sets. Under another correlate perspective, Justin Lundberg defines voice leadings in post-tonal contexts "as ordered sets of individual pitch-class mappings from one pitch-class set to another" (Lundberg 2012, p. 2). From this assumption, he proposes a theory of *voice-leading sets* (vlsets), which "express ordered sets of

meets Tymoczko's basic methodological intention of systematization of the subject, in spite of both being substantially distinct in content.

pitch-class transpositions or inversions as vectors" (: 3). These, in turn, are organized in *voice leading set classes* (vlclasses), which present, analogously to pitch-class sets, equivalence under transposition and inversion operations.

In a recent study, Sean Smither 2019 proposes a theory of guide tones, a well-known concept by jazz improvisers. His *guide-tone space* (GT space) addresses melodic connections of 3-7 dyads in jazz harmonic progressions through use of a formal apparatus derived from Lewin's transformational theory, resulting into algebraic and graphic representations. As it will be evidenced, Smither's approach has some interesting affinities with the idea of c-PVLs which is here developed.

2. Basic concepts

This study assumes as initial and essential premise that objects (chord notes) and relations (their individual connections) are projected on the *pitch space* (according to the well-known concept coined by Robert Morris 1987).[9] Concepts and propositions that are described in the next subsections are firmly grounded on this basic assumption.

2.1 Binary relations

Firstly, let the ordered duple (C_1, C_2) be defined as a *binary chordal relation*, or simply a binary relation, between two contiguous chords, C_1 and C_2. Let also k_1 and k_2 be the variables that represent the respective *cardinalities* of these chords, that is, the number of notes (including eventual doublings) that form them. Define a *pure* binary relation that in which $k_1 = k_2$. A *hybrid* relation, in turn, admits two possible situations, namely $k_1 > k_2$ and $k_1 < k_2$.

Figure 1 depicts some examples of binary relations. It is worth to notice that in the present context cardinality does not mean necessarily the same as chordal complexity. In (a) a pure relation of cardinality 4 involves a triad (with fifth duplicated) and a ninth chord (in which the fifth was omitted). On the other hand, in (c) two seventh chords are connected by a hybrid relation. In sum, the present focus is centered on the relations of the voices that form the chords rather

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[9] In opposition to the *pitch-class space* (pc space), formed by only the twelve pitch classes.

than on the internal chordal structure. The implications of this perspective are examined in the next subsections.[10]

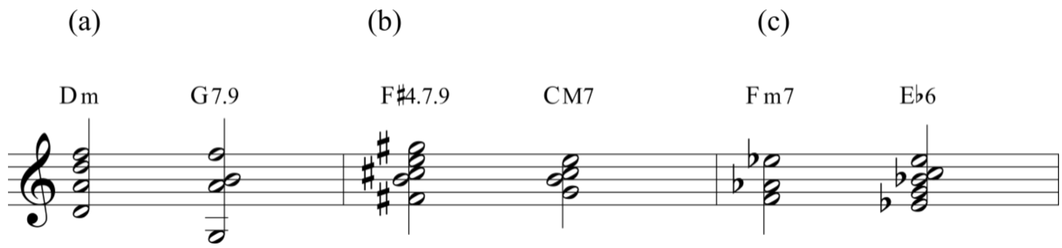

Figure 1: Examples of binary relations: (a) pure ($k_1 = k_2$); (b) hybrid ($k_1 > k_2$); (c) hybrid ($k_1 < k_2$).[11]

2.2 Voice linkages

The notion of *voice linkages* will here refer to the types of connections between the individual voices that are present in a binary relation.[12] Linkages can be: *paired* (PR), a simple, one-to-one relation; *convergent* (CV), when two voices merge towards one; and *divergent* (DV), when a voice splits into two others.[13]

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[10] Hon Ki Cheung 2018 addresses the problematic issue that inherently arises from smooth voice leading in the connection of chords with different cardinalities (in her case, involving triads and seventh chords). This question is also a topic of interest for Matthew Santa 2003, in the study of voice leading in *nonatonic systems*, taking as case studies some compositions by John Coltrane (like "Giant Steps"), whose harmonic structures alternates major triads and dominant seventh chords. Santa deals the question with two streams of idealized voicing: in the higher, three voices are smoothly connected, omitting the fifths of the dominants which are included in the bass line that progresses downwards as a whole-tone scale.

[11] The notation adopted for chord labels is based on criteria presented in Almada (in preparation).

[12] The term "voice linkage" was chosen to differ from the more general notion of "voice leading", which can be used for denoting movements both between pitches or pitch classes. A voice linkage, in this study, refers to the individual connection of two same-stream pitches considering their specific register (in this manner, $C_4 \rightarrow A_4$ is a voice linkage distinct from $C_4 \rightarrow A_5$ or $C_4 \rightarrow A_3$, etc.). Along the article both terms will be employed, according to the general (voice leading) and specific (voice linkages) senses which are intended.

[13] A number of theorists have already explored similar typologies of melodic connections. Clifton Callender 1998 adopts the terms "split" and "fuse" for addressing voice leading of sets with incompatible cardinalities. Operations of splitting and merging voices (mandatory in the connection of unequal-cardinality chords) are also proposed by Tymoczko 2011, considering them also in the formalization of transformations between scales (or "macro-harmonies", in his terms). Commenting about Douthett and Steinbach's τ function (1998), used for depicting parsimonious relations between two pc sets of same cardinality, Cheung 2018 expresses some concerns about its incapacity of working with hybrid situations (triads and seventh chords, for example), a central point of her article. According to the author, this problem is better treated by Joty Rockwell 2009, with his *P-matrix* (see more details in footnote 24). The notions of DV and CV

Figure 2 presents the three types using a schematic representation. Paired linkage is the most common configuration, being typical of pure-relation cases (a). Convergent linkage is necessary for situations of type $k_1 > k_2$ (b), and the divergent type for the opposed case (c). However, there is always room for different interpretations, especially in ambiguous situations. As an illustration of this issue, Figure 2.d proposes an alternative configuration for the scheme of Figure 2.a.

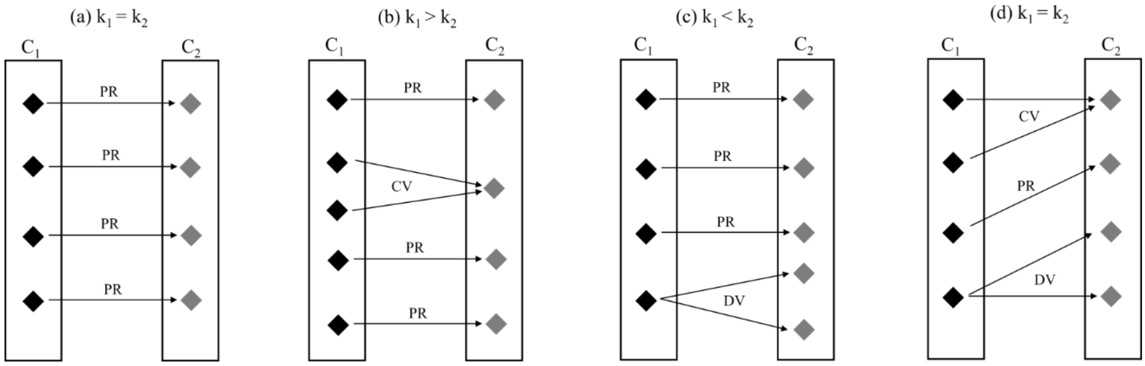

Figure 2: Examples of voice linkages, considering the three possible types of binary relations: $k_1 = k_2$ (a; d), $k_1 > k_2$ (b), and $k_1 < k_2$ (c).

Before discussing how to deal properly with situations like these, it is important to examine also the voice linkages according to their intervallic spans, a necessary refinement for the present theory. The linkages can be classified into three simple categories:[14]

parsimonious: refers to intervals that are lesser than or equal to a major second, or two semitones (this category includes the "0-semitone" interval, or unison);

skip: refers to intervals that are greater than a major second and lesser than or equal to a perfect fourth, or five semitones (informally, skips are associated with arpeggio-like and broke-chord movements);

leap: refers to intervals that are greater than a perfect fourth.

here proposed shall not be seen as simply new designations for the same concepts. In fact, they are mainly intend to denote connections mapped on the pitch space, involving specific voicings (and not idealized ones, as normally considered in theory), matching the idea of “voice linkages”.

[14] This typology is roughly based on the classification proposed by Jay Dowling 1978.

Voice-linking efficiency (in the terms to those proposed by Tymoczko) in a harmonic progression is strongly dependent on a greater proportion of parsimonious linkages in comparison with the other types. More specifically, parsimonious intervals of *minimal disturbance* (i.e., unison and minor seconds) are the main actors if highest possible efficiency is aimed. Skips, associated with arpeggio-like movements, affect negatively the overall smoothness of a voicing, being relatively rare in the covered repertoire. Leaps are typically left to the connection of the basses of the chords, especially if these are written in root position (which is normative in popular-music harmony).[15]

2.3 Preference rules

Considering that a preliminary evaluation of how the voices of a harmonic progression can be connected is paramount for a voice-linking analysis, it becomes indispensable, before continuing, to establish some *preference rules*[16] for the choice of the best interpretation, facing a set of possible alternatives.

The rules are in number of five, and are primarily elaborated according to the principles of logic, economy, and simplicity,[17] being expressed as it follows:

1. **Prefer configurations in which all voices of both chords are linked** – this rule departs from the premise that the chords of a progression are formed by consistent contrapuntal lines. Although eventually a line may be interrupted or a new one initiated, these cases must be considered as quite exceptional;
2. **Prefer paired linkage where possible** – this rule is primarily intended to favor the maximization of paired movements in hybrid binary relations. Besides this, it inhibits, all being equal, non-paired configurations like that

[15] As it will be observed in the analysis, Jobim minimizes the presence of leaps in the bass by the use of parallelism in harmonic connections, especially chromatic and descending.

[16] The idea of a system of rules of preferences is inspired by those proposed by Fred Lerdahl and Ray Jackendoff 1983 and, as before mentioned, by David Huron 2016, in this case specifically addressing voice-leading questions.

[17] Although some rules may appear to overlap or be redundant (or even unnecessary), their content and order were carefully designed in order to properly deal with complicated, ambiguous situations, involving multiple (and sometimes conflicting) interpretations. The formation of the current version of this rule system was preceded by many tests through the use of a computational algorithm especially designed for this task. Several pre-existent rules were simply eliminated, others merged into new ones. The final group was consolidated when all experimental, problematic cases before imagined were adequately addressed with the rules.

exemplified in Figure 2.d (unless strong reasons justify them). In all cases, the application of this rule is conditioned by the two next ones;

3. **Prefer strongly configurations without crossing of voices** – this rule corresponds to the well-known principle traditionally adopted in the pedagogy of harmony. Although in the present context the idea is to work with relatively independent melodic lines, indiscriminate crossings tend to contribute to turn obscure the logic and clarity of the individual streams;[18]
4. **If a leap is necessary, prefer to use it in the lowest voice** – this rule conditions situations where a given maximally-paired configuration implies the adoption of a leap in a non-bass voice. In these cases, the leap shall be transferred to the bass, forcing the use of divergent or convergent linkages in the upper voices;
5. **In the cases where divergent or convergent linkages are inevitable, prefer the most economic possible configuration** – this allows for bypassing rule 2 (preference for pairing linkages), if a more economic configuration is available. The degree of economy of a divergent or convergent linkage is evidenced by the absolute size of the two intervals involved in the movements.

2.4 Plotting voice linkages

Some examples will illustrate the use of the rules in the choice of the optimal voice linking for binary relations. For a better visualization of the voicing connections, a simple pitch-space graphic model (called *pitch-column graph*) was idealized, complementing score representation (Figure 3). The two columns correspond to the chords, with the squares representing semitones. Activated squares indicate the involved chordal notes, and the lines connecting them evidence the linkages selected out of the three types of intervallic motions

[18] This rule was based on David Huron's Preference Rule 14, which determines the avoiding of crossing melodic parts. As the author explains, "is impossible for simultaneous parts to cross without some disruption of the auditory streaming. So, what happens when the pitches of two sound sources actually do cross? Lines are perceived to switch direction at the point where their trajectories cross. Listeners hear the lines as bouncing away from each other rather than crossing. Grouping all the high pitches together (and grouping all the low pitches together) is more important than preserving a consistent rhythmic pattern. Pitch-based streaming takes precedent over rhythmic regularity" (Huron 2016, p. 41–42). Tymoczko (2011, p. 90 ff.) expresses similar concerns about crossing voices, naming *crossing-free voice leadings* the ideal situations, aiming at optimized efficiency.

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(parsimonious, skips, or leaps). Initially, Figure 3 illustrates a case of pure binary relation, with three alternative linkage configurations.

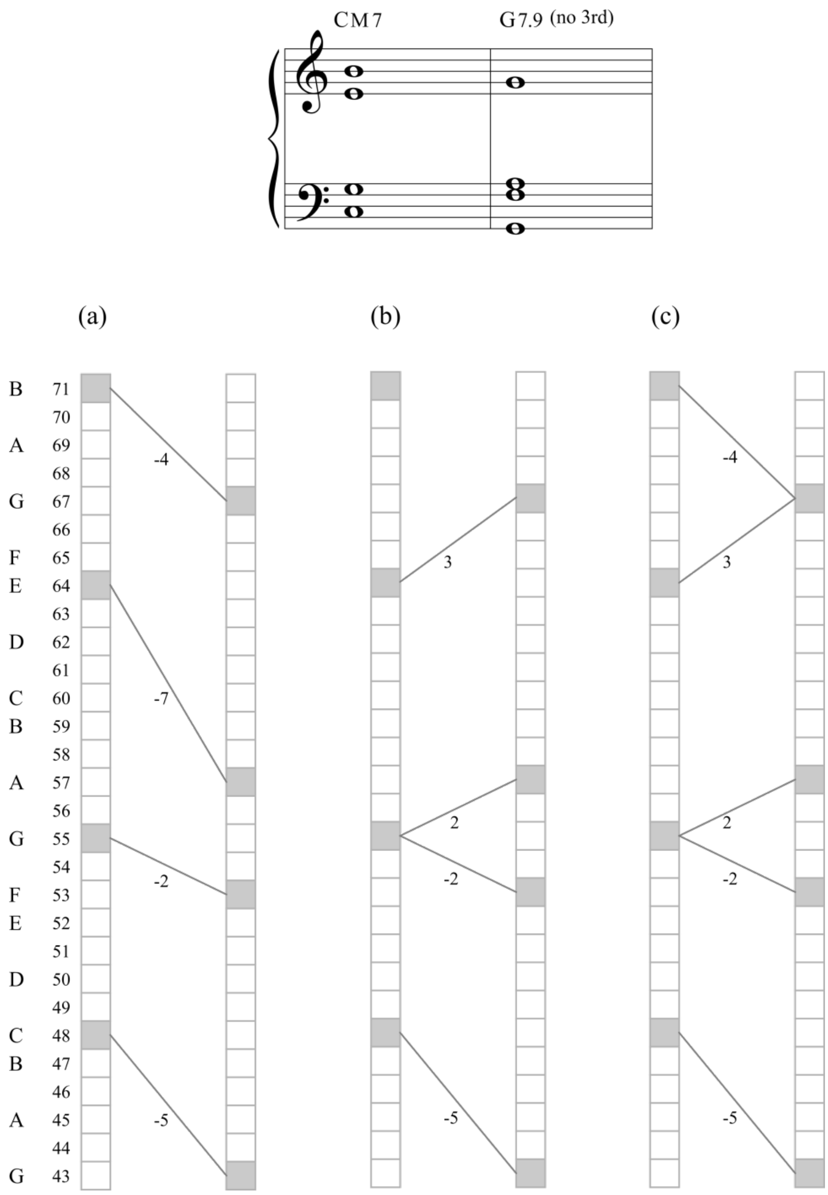

Figure 3: Alternatives of voice linking in a given pure binary relation ($k_1 = k_2$). Edges indicate intervallic distances in semitones.[19]

[19] This spatial model of voice-leading analysis adopts a similar logic of the methodology employed by Jonathan Bernard 1994 in his study on some of Ligeti's compositions.

Option (a) contemplates rule 2 (all voices shall be paired in some way), but this results in a leap between E_4 and A_3, which conflicts with rule 4 (leaps shall be confined to the bass line). Option (b) fixes this problem by letting the voice leaded by G_3 to split into two parsimonious linkages, to F_3 and A_3, and making G_4 to be reached by E_4. This solution is not entirely adequate, however, since the highest line is stopped, breaking rule 1 ("all voices shall be connected"), which makes option (c) the best alternative for the situation.

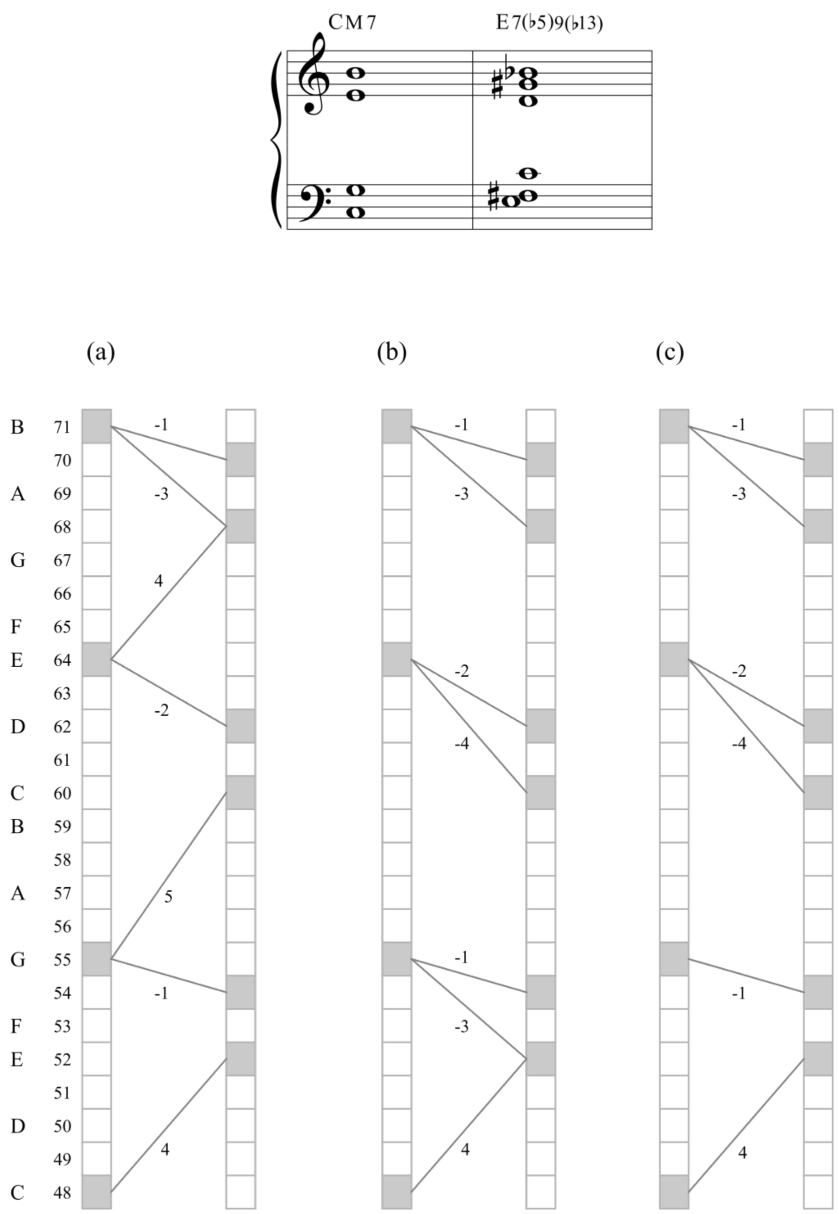

Figure 4: Alternatives of voice linking in a given hybrid binary relation ($k_1 < k_2$).

Figure 4 exemplifies a case of hybrid relation, type h2. The three alternatives proposed contemplate rules 1, 2, 3, and 4. Their differences lie therefore on the issue of divergent connections (rule 5). In (a) the note G#$_4$ is unnecessarily approached by two divergent linkages of different sizes. Moreover, note C$_4$ of the second chord is reached by a skip of perfect fourth, while a more logical solution is available, as shown in option (b). Always guided by simplicity and economy, it is easy to perceive that the divergent linkages from G$_3$ into F#$_3$ and E$_2$ are superfluous. The preferred configuration is presented in (c).

A third example (Figure 5) considers a binary hybrid relation of type $k_1 > k_2$. In this case, a convergent linkage is necessary. Option (b) provides the most compact alternative in intervallic terms.

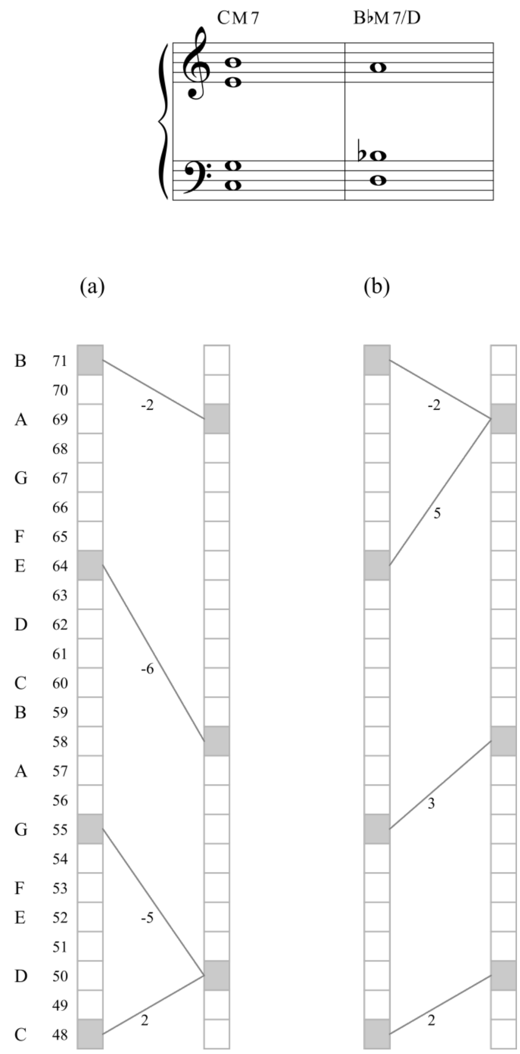

Figure 5: Alternatives of voice linking in a given hybrid binary relation ($k_1 > k_2$).

Besides the analytical application aimed in this proposal, knowledge about economic voice linking can be very useful in other musical domains, like arranging and performance. Regarding this latter aspect, pianists reading chord labels intuitively prefer to connect harmonies oriented by the "least-effort" principle, that is, keeping common notes and moving others by minimal possible distances (in other words, by using informally voice-leading preference rules, like those above formulated), which optimizes real-time performance: by touching near keys with his/her fingers (eventually keeping some of them immobile) it turns unnecessary (or at least minimally necessary) for the pianist to deviate the eyes from the score to the keyboard.[20]

3. Parsimonious voice leading (PVL)

Let us now consider voice-leading binary relations in terms of only parsimonious motions, or else, disregarding eventual skips and leaps.[21] This strategy aims at evidencing the presence of special patterns of melodic connections, related to both intervallic content and relative motion between the involved linkages. As a basic hypothesis, considering the covered repertoire in this theory, some patterns are more recurrent than others, due not only to the smoothness of the connections, but also to special compositional strategies.[22]

Let us name these relations *parsimonious voice leading*, identified from now on with the acronym PVL. A PVL can comprise a selected subset of the voicing of a binary relation (i.e., excluding skips and leaps from the analysis) or, in some

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[20] This argument is supported by practical instruction for jazz pianists in respect to how the voicings of chord progressions shall be constructed. For some illustrations and specific commentaries about the voicings of different harmonic situations typical in jazz music (like cadences, “two-five” variants, passing chords, tritone substitution, etc.) see, for example, Mark Levine (1989, p. 137–154).

[21] In a similar approach, Tymoczko 2011 coined the concept of *nonfactorizable voice leadings*. This is clearly represented by his "3+1" scheme, in which a bass line of a chordal progression is considered separately from economical configurations of the superior voices. Although the bass line generally is connected by skips or leaps in the context of popular-music harmonies, this is not always the case in some of Jobim's progressions (some examples in this article illustrate this claim).

[22] This argument will be resumed at the end of the article.

special cases, the whole voicing. Firstly, define PI as the set of parsimonious intervals, measured in semitones, such that PI = {-2, -1, 0, +1, +2}.

Figure 6 introduces an intuitive graphic representation of the five types of parsimonious intervals.[23] Aiming to facilitate visual identification, edges differ in style according to the magnitude of the interval between nodes: dashed (whole tone), filled (semitone), and dotted (unison). In this manner, the indication of intervals becomes redundant and will be suppressed in future examples.

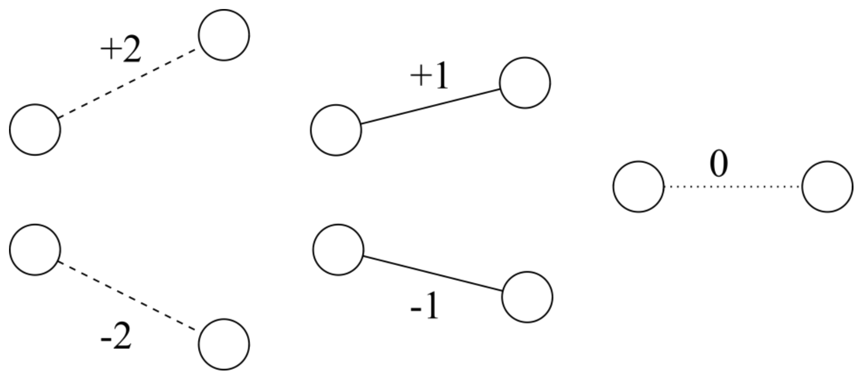

Figure 6: The five parsimonious intervals in graphic representation.

Now, define the *parsimonious vector* Γ as the ordered sequence of entries

$$\Gamma = <[-2][-1][0][+1][+2]>$$

where the numbers inside brackets represent instances of the respective parsimonious intervals present in a given binary relation.[24] Vector Γ is used to

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[23] This kind of representation favors an alternative, transformational perspective for this study (in the sense proposed by Steve Rings 2011, in which notes of a chord do not properly "move" to their following dispositions in the second chord (as the conventional approach of voice-leading studies), but rather are *transformed* into other notes. Furthermore, since symmetric organization is strongly present in these special parsimonious connections, it is also possible to treat them as forming an *algebraic group*. In spite of the attractive implications that these new views arouse, for the sake of convenience and simplicity, the traditional metaphor of movement of voices embedded in harmonic progressions will be maintained along this article, and the group/transformational potentialities of this theory will be left to future exploration.

[24] The structure of this vector can be compared with that of the *P-matrix*, proposed by Joti Rockwell 2009. It corresponds to a square matrix 2x2 in the form $\begin{pmatrix} u1 & u2 \\ d1 & d2 \end{pmatrix}$, where the pairs "u1/d1" and "u2/d2" denote the occurrences of, respectively, semitone and whole-tone movements in a given voice leading involving two pc sets. Unlike vector Γ, however, Rockell's matrix does not inform the presence of note retentions (consequently, the precise number of relations involved cannot also be determined). Another similar construct is Santa's PVLS (*parsimonious voice-leading sum*), elaborated for "measuring the degree of parsimonious of any voice leading" present in his analysis of Coltrane's pieces. As explained by the author, the function PVLS applied to two pc

identify a PVL according to its specific parsimonious properties, analogously to the manner an interval-class vector (icv) is associated with a specific pitch-class set.[25]

PVLs related to binary chordal relations are constructed through combination of units (or blocks) composed by two or three motions by parsimonious intervals. The main idea behind the gathering of individual parsimonious linkages in blocks is to try to capture high-level patterns of voicing organization (involving the interaction of relative-movement categories, as contrary, similar, parallel, and oblique), especially those more common in the aimed repertoire. This will be elaborated along this and in the next section.

Algebraically, a PVL is identified by the ordered duple (x, y), where x and y represent respectively the quantities of two- and three-voice blocks that integrate it. PVLs can be *simple* (when either x or y equals zero) or *compound*. A PVL is said *unitary* if the highest value among x and y is 1, otherwise it will be classified as *multiple*. The number of voices *n* that act in a PVL is calculated by the formula

$$n = 2x + 3y \tag{1}$$

Table 1 depicts a PVL typology, considering situations from two up eight parsimonious voices.

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sets X and Y maps "the amount of voice-leading motion expended in one direction, measured in semitones", in the form "PVLS(X, Y) = $|int_1$ (the interval between the first elements of X and Y), $int_2, \ldots, int_n|$" (Santa 2003, p. 15–16). By its turn, a *parsimony vector* (PV) in Santa's theory is an ordered triplet that, when applied to a pair of two chords (X, Y), informs the PVLS, the number of half steps (HS), and the number of whole sets (WS) between two same-stream notes that the voice leading contains. This is formally presented as PV(X,Y) = PVLS(X,Y), HS(X, Y), WS(X, Y). As an illustration of this procedure, the author depicts the PV of the idealized voice leading between the triads of C+ (C major) and A≅– (A≅ minor) : PV(C, A≅–) = $|-1 - 1 + 1|$, 3, 0 = 130. However, Santa's system is projected to deal only with sets with compatible cardinalities (fifths are omitted in seventh chords when related to triads).

[25] Like usual icv's algebraic representation, the number of instances inside Γ, from now on, will be depicted inside angled brackets, without separating commas.

x	y	type	n (2x+3y)
1	0	simple-unitary	2
0	1		3
2	0	simple-multiple	4
1	1	compound-unitary	5
0	2	simple-multiple	6
3	0		
2	1	compound-multiple	7
4	0	simple-multiple	8
1	2	compound-multiple	

Table 1: PVL's structure for different number of voices (from two to eight).

A possible argument contrary to this system of organization of PVLs could be it is both complicated and arbitrary. Why not consider blocks simply as they appear in musical situations, forming two linkages or three, six, and so on? Which is the special reason for factoring PVLs by 2 and 3? Why not 4, for example? These are reasonable questions that can be answered as it follows: the idea of filtering the large universe of possible textural cases in a pair of basic categories is motivated by an intention of avoiding a huge proliferation of types, which could result into an increased difficulty (or even unfeasibility) for a proper classification of PVLs. On the other hand, the use of binary and ternary blocks (and not with four or more elements) for this task is due to main two reasons. Firstly, it allows that the simplest, non-unary voice-linkage situation (the connection of a pair of voices acting in a binary chordal relation) be mapped with the same methodology used for higher-density cases (this would not be possible with a fixed four-block pattern, for example). As a second reason, the present system was idealized in such a way that any possible number of linkages can be described in terms of the two most basic co-primes, 2 and 3 (as suggested in equation (1) and Table 1), which represent the PVL's basic building blocks. This provides the model of concision, simplicity and breadth. In addition, some specific rules for parsing the linkages into two- and three- blocks are intended to constrain the possibilities and eliminate conflicting alternatives. These rules will be explicated in due time.

PVLs are graphically represented as *oriented networks*,[26] through combination of the parsimonious intervals. So, by gathering basic "atoms" (the PIs), PVL "molecules" with different degrees of complexity can be formed, as exemplified in Figure 7.

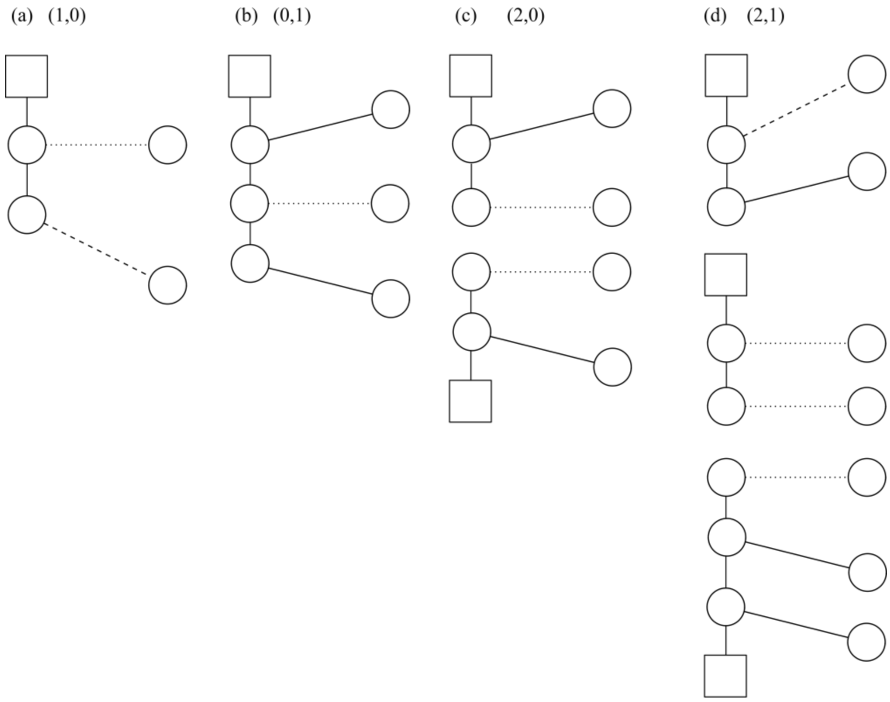

Figure 7: Representation of PVLs related to binary relations of different sizes: (a) two; (b) three; (c) four; (d) seven. Squares and vertical lines group PIs and form a PVL. Vertically aligned PVLs (as in *c* and *d*) denote a compound binary relation.

PVLs shall be seen as abstract representation of melodic connections (that may be positioned in many distinct ways in a score). Put simply, the only relevant information that they convey are the *intervallic and directional relations* between the voices, rather than their actual registral positions. From this, we may conclude that a PVL is an unordered configuration of parsimonious intervals and, therefore, it is equivalent to any permutation of itself. Figure 8 depicts a particular (0,1) PVL, formed by three distinct PIs (+1, -1, and 0), flanked by five

[26] This means that the edges denote the left node is followed by the right one (or transformed into it). Since it is assumed that the events represent melodic motions across time, all connections must be read from left to right (or else, retroactive interpretations are out of question in this case), which turns unnecessary, in the present context, the conventional use of arrows to indicate the direction of movements. For a more detailed discussion about oriented networks, see Rings (2011, p. 101–148).

other re-arranging of the same content. Aiming at a padronization, define a *prime form* of a PVL[27] as the unique of its possible representations that respect a fixed top-down order, namely ascending-unison-descending, according to the respective intervallic magnitude (i.e., -2, -1, 0, +1, +2), as is the case of highlighted alternative of Figure 8.a. Given this, from now on, and all things being equal, PVLs will be preferecially represented by their prime forms.[28]

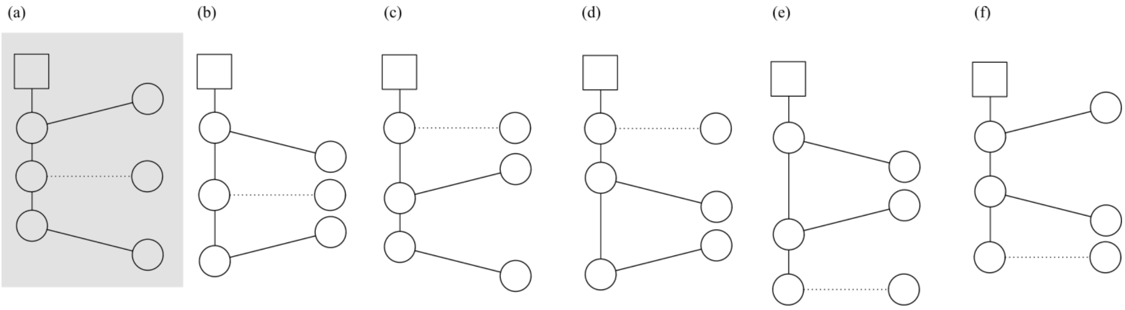

Figure 8: A particular PVL prime form (a) and five equivalent configurations (permutations) of it (b-f).

Let us now start the process of qualification of PVLs, by examining systematically those which are *simple* and *unitary*, that is, types (1,0) and (0,1). Being both the building blocks for the construction of any possible more complex parsimonious voice-leading configuration, the elaboration of an exhaustive taxonomy of their prime forms can be seen as a necessary theoretical stage.

PVL (1,0) is formed by a pair of voices and, consequently, two melodic intervals, i_1, $i_2 \in PI$. These intervals can relate one to another in two unique basic manners, namely $i_1 = i_2$ and $i_1 \neq i_2$. Let these two possibilities be called *PVL classes*, labeled, respectively, as *a* and *b*. Considering that there exist five possible states for i_1 and i_2, and that, as previously stated, the order in which are disposed is not relevant, we can obtain, by the use of simple combinatorics, five members in class **a** and ten members in class *b*, as depicted in and exemplified in Figure 9.[29]

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[27] Analogously to what is adopted in Pitch-Class Set Theory, a PVL prime form represents all possible re-configurations of its content (in this case, a particular subset of PI), to which mantains equivalency under operation of permutation.

[28] As it will be seen in section 5, the use of prime forms is not always possible in analysis, due to spatial constraints and contextual conditions, which justifies the eventual relaxing of this rule.

[29] This strategy is inspired on the central idea behind the Theory of *Partitional Analysis*, created by Pauxy Gentil-Nunes, which is in turn based on theoretical formulations by Leonhard Euler.

PVL class	i_1	i_2	Γ	member
a ($i_1 = i_2$)	–2	–2	<20000>	a1
	–1	–1	<02000>	a2
	0	0	<00200>	a3
	+1	+1	<00020>	a4
	+2	+2	<00002>	a5
b ($i_1 \neq i_2$)	–2	–1	<11000>	b1
		0	<10100>	b2
		+1	<10010>	b3
		+2	<10001>	b4
	–1	0	<01100>	b5
		+1	<01010>	b6
		+2	<01001>	b7
	0	+1	<00110>	b8
		+2	<00101>	b9
	+1	+2	<00011>	b10

Table 2: PVL classes *a* and *b* (1,0) and respective members.

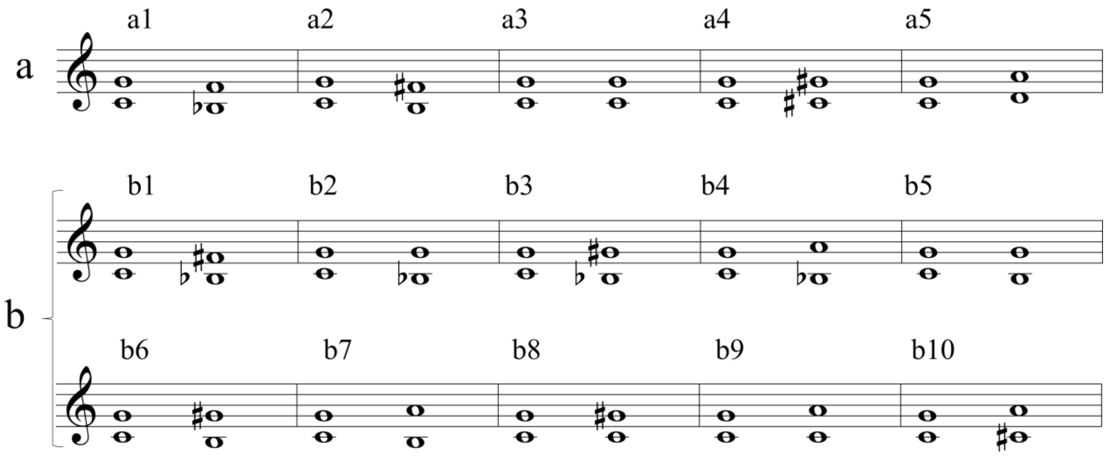

Figure 9: Examples of PVL classes *a* and *b*.

PVL (0, 1) is formed by a block of three voices and, consequently, three melodic intervals, i_1, i_2, $i_3 \in$ PI, which can relate one to another by three distinct manners, namely $i_1 = i_2 = i_3$, $i_1 = i_2 \neq i_3$, and $i_1 \neq i_2 \neq i_3$. Let us call them classes *c* (five

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Basically, it refers to the property of an integer to be expressed as a sum of other integers. For a comprehensive description of the elements of the Partitional Analysis, see Gentil-Nunes 2018.

possible combinations), *d* (twenty possible combinations), and *e* (ten possible combinations), respectively (see Table 3 and Figure 10).

PVL class	i_1	i_2	i_3	Γ	member
c ($i_1 = i_2 = i_3$)	–2	–2	–2	<30000>	c1
	–1	–1	–1	<03000>	c2
	0	0	0	<00300>	c3
	+1	+1	+1	<00030>	c4
	+2	+2	+2	<00003>	c5
d ($i_1 = i_2 \neq i_3$)	–2	–2	–1	<21000>	d1
			0	<20100>	d2
			+1	<20010>	d3
			+2	<20001>	d4
	–1	–1	–2	<12000>	d5
			0	<02100>	d6
			+1	<02010>	d7
			+2	<02001>	d8
	0	0	–2	<10200>	d9
			–1	<01200>	d10
			+1	<00210>	d11
			+2	<00201>	d12
	+1	+1	–2	<10020>	d13
			–1	<01020>	d14
			0	<00120>	d15
			+2	<00021>	d16
	+2	+2	–2	<10002>	d17
			–1	<01002>	d18
			0	<00102>	d19
			+1	<00012>	d20
e ($i_1 \neq i_2 \neq i_3$)	–2	–1	0	<11100>	e1
			+1	<11010>	e2
			+2	<11001>	e3
		0	+1	<10110>	e4
			+2	<10101>	e5
		+1	+2	<10011>	e6
	–1	0	+1	<01110>	e7
			+2	<01101>	e8
		+1	+2	<01011>	e9
	0	+1	+2	<00111>	e10

Table 3: PVL classes *c*, *d*, and *e* (0,1).

Figure 10: Examples of PVL classes *c*, *d*, and *e*.

An alternative (and analytically useful) way of arranging the five classes is by associating them with the traditional types of relative movements between voices, namely parallel, similar, oblique, and contrary. To these four types it is added a fifth, *identity*, that is, a perfectly static voicing. Only two cases are representative of this type: a3 and c3, two- and three-voice identity, respectively. All remaining members of classes *a* and *c* belong to the category of parallel movement (in a total of eight instances). Similar-movement situations combine two different non-zero intervals in same direction (there are six instances in this category). Oblique-like cases are characterized by the presence of one or two unisons (depending on the number of acting voices), plus one or two non-zero, non-parallel intervals (i.e., ±2 or ±1) in the same direction (fifteen instances).[30] The

[30] In other words, it is necessary that the presence of movement(s) in a *unique* direction (up or downwards, performed by either one or two voices), against one or two stationary voice(s).

remaining class members necessarily belong to the contrary-movement category, the most numerous (nineteen instances).

Table 4 depicts this alternative organization of the PVL classes.

Identity (i)	Parallel (p)	Similar (s)	Oblique (o)	Contrary (c)
a3	a1, a2, a4, a5	-	-	-
-	-	b1, b10	b2, b5, b8, b9	b3, b4, b6, b7
c3	c1, c2, c4, c5	-	-	-
-	-	d1, d5, d19, d20	d2, d6, d9, d10, d12, d15, d16, d17, d18	d3, d4, d7, d8, d11, d13, d14
-	-	-	e1, e10	e2, e3, e4, e5, e6, e7, e8, e9

Table 4: PVL member classes grouped according to the five categories of relative movement of voices.

Some examples in network notation are presented in Figure 12. Observe that the corresponding labels are inserted in the squares, identifying precisely the PVLs, a convention that will be adopted from now on.

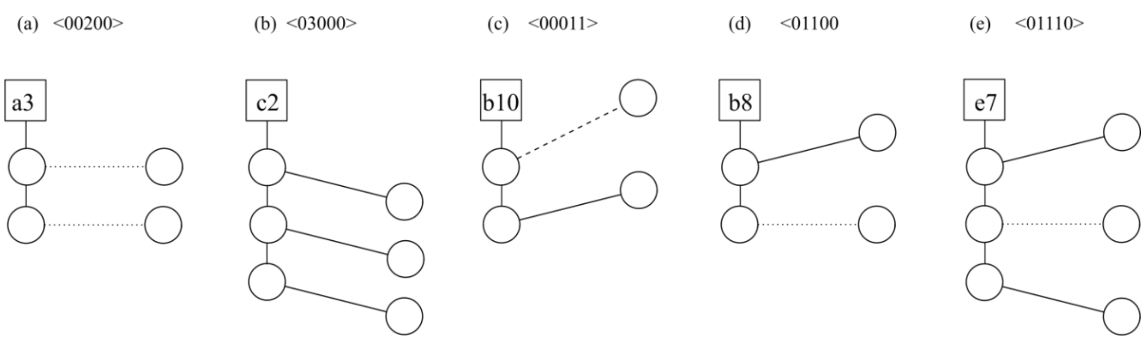


Figure 11: Examples of PVLs associated with the five relative-movement categories: (a) identity; (b) parallel; (c) similar; (d) oblique; (e) contrary. Labels inside the squares identify the respective class members.

4. Contextualized PVLs

By combining contrapuntal information regarding movement of voices (conveyed by PVL classes) with harmonic structure of the chordal qualities involved in a binary relation, it is possible to obtain a robust and more refined tool for qualifying parsimonious voice-leading patterns. These combined structures are called *contextualized PVLs*, or c-PVLs, for short. In network representation of c-PVLs, the nodes, so far let empty, will be assigned to *functional labels* of the chordal notes which represent. The functional label of a given note is intended to depict the role that it plays in the chord it is inserted in, namely root (symbolized by number 1), third (3), fifth (5), and so on. Figure 12 provides a simple illustration of an analytical application of c-PVLs. As shown in the example, the same melodic voicing pattern depicted by the oblique-like class b5 may have different *functional meanings* depending on the harmonic framework considered (evidently, many other possibilities could be also added to the example). In other terms, it is easy to perceive that any abstract PVL class can potentially become the basis for a considerably huge amount of c-PVL realizations.

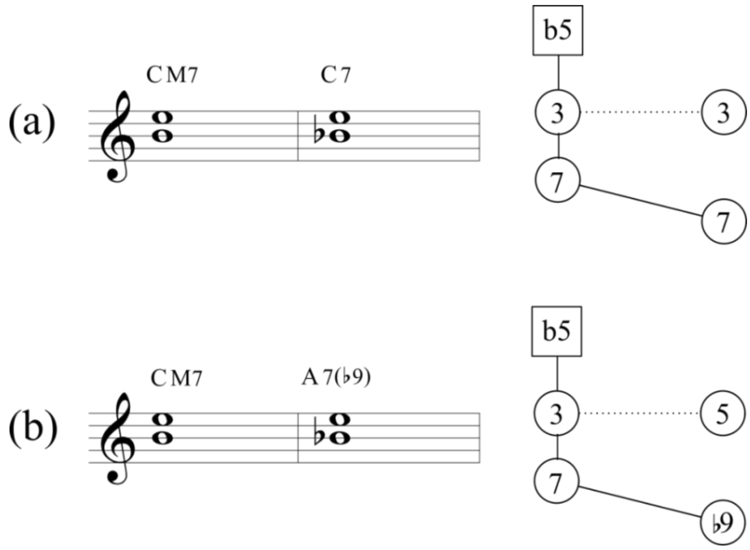
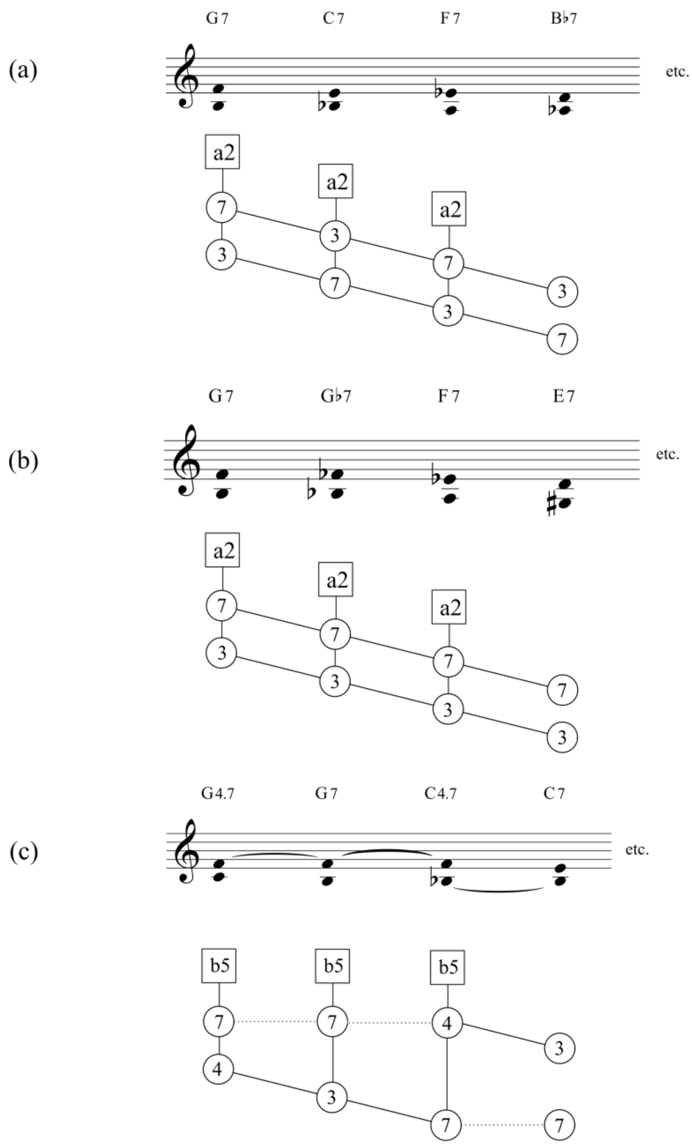


Figure 12: Two distinct instances of c-PVLs obtained from different harmonic realizations of PVL class b5.

Some c-PVLs animate common voice-leading patterns, like those associated with chains of secondary dominants. As commented by Walter Piston in his textbook "Harmony" (1987, p. 262–263), a very effective manner for treating contiguous seventh dominants is to voice them in a "barber-shop" manner, or else, replacing tritone normal resolution (7→3/3→1) by chromatic move in both voices (7→3/3→7). Figure 13a depicts this procedure as a series of a2s in which sevenths and thirds are alternated in both lines. If tritone substitution (commonly known as "subV chords") is applied to alternate dominants a complete parallelism of functional notes is achieved inside the same PVL configuration (b). The use of suspended-fourth dominants is quite idiomatic in jazz music. In terms of voice-leading behavior, this device dilutes the pure parallelism and gives a somewhat "tectonic" effect to the chromatic dislocations, which is captured by a series formed by PVLs b5 (c). Extensions are normally added to these patterns in order both to turn denser the sonorities and to improve the voice leading. There are many possible variants used in jazz music, one of them is exemplified in (d). Observe how PVLs d6 and d10 (both related to b5 by inclusion of distinct PIs) alternate as one or two voices are retained.



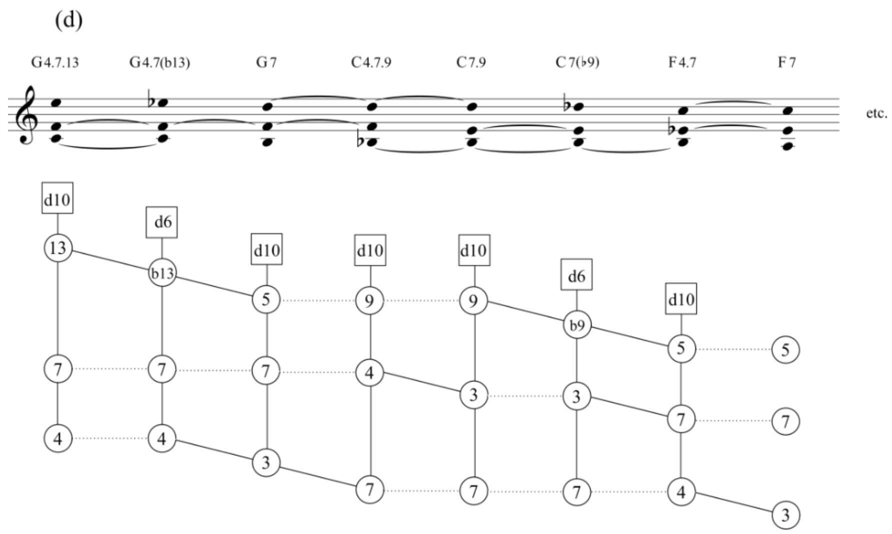


Figure 13: c-PVLs in consecutive dominants.

Interestingly, the actions of neo-Riemannian operations can be modeled by three-voice c-PVLs. As shown in Figure 14, under this perspective it is possible to consider operations P (parallel) and L (*Leittonwechsel*) applied to a major triad as particular (or, better, *contextualized*) realizations of same PVL class member d10, in which just the note-functional patterns differ (other operational outputs are also exemplified in Figure 14). Considering the scope of c-PVL theory (jazz, bossa, and co-related subgenres), a repertoire in which parsimonious voice leading rarely involves triadic structure, some of these neo-Riemannian-like configurations can be "inhabited" by notes of more remote chordal relations (mostly harmonic extensions) inside PVL more complex "molecules".

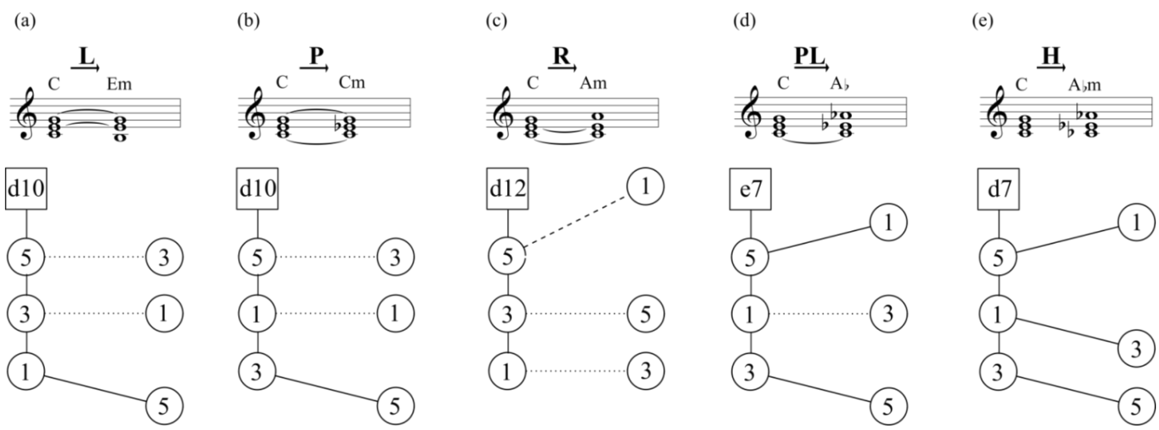


Figure 14: Some neo-Riemannian operations represented as contextualized PVLs: (a) d10/L; (b) d10/P; (c) d12/R (Relative); (d) e7/LP; (e) d7/H (Hexatonic pole).

The application of this technology in the analysis of real music can now be introduced. An initial, common difficulty faced by an analyst concerns the basic convention that determines that c-PVLs should preferentially be graphically represented in prime forms. As elsewhere mentioned, this orientation is not always possible to be respected, due to the manner that which the binary relations are particularly organized in a given harmonic progression. This obstacle can be intensified by the presence of divergent and/or convergent linkages between voices. Figure 15 illustrates this problematic issue with the help of a simple example. This short harmonic progression involves two binary relations, represented by a sequence of two three-voice PVLs: e7-e3. This sequence is plotted in (a) according to the rule of prime-form representation. In order to keep this normative disposition in both PVLs, however, it would be necessary to use an "intermediary zone" (shaded in the example) for re-adjust the positions of the nodes (see the dashed-dotted edges). Moreover, divergent/convergent movements should be "rectified" through duplication of the fused voices (1 in CM7 and 5 in A7) on the network. These unnecessary complications can be overcome with a more concise notation for the connections (b).[31] This shortcut is supported by the principle of equivalence between any possible realizations of a c-PVL, provided that the relations expressed by vector Γ, as well as the positions of chordal labels are preserved. In other words, the constraint regarding to the preferential use of prime forms, from now on, may be relaxed, depending on contextual conditions.

[31] Occasionally, adjustments on the size of the vertical lines that group the motions must also be made (as in the case exemplified), in order to allow for accommodating fusions.

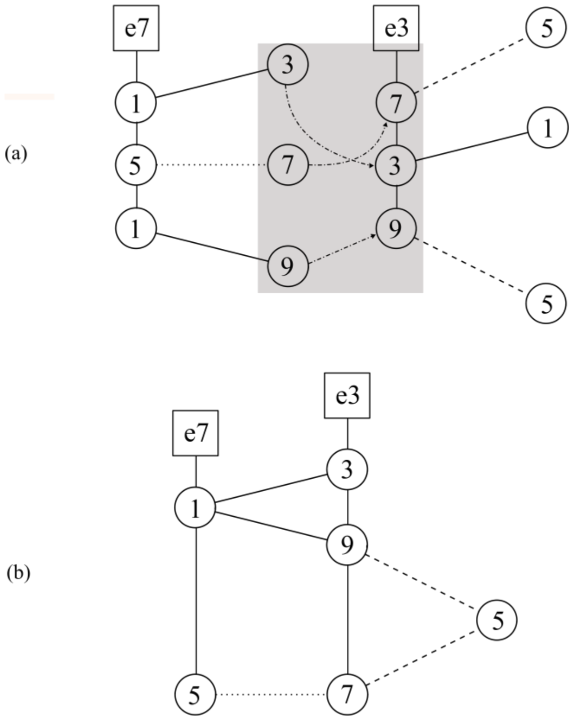


Figure 15: Example of c-PVL analysis using two equivalent network notations: with prime forms (a) and concise (b).

Another pertinent question concerns how to group parsimonious connections in PVL blocks. Although, as known, any permutated configuration can be considered equivalent, the adoption of precise criteria for parsing the voices seems to be, at least, a reasonable measure in itself, aiming at standardization. As a first convention, c-PVLs shall preferentially be formed by observing the registral order of the involved notes, as depicted in the score. For each binary relation to be analyzed, the segmentation of voices into blocks (of two or three components) shall respect the following guidelines (see Figure 16): if the number of parsimonious voices is lesser than five, use the expected, appropriate configuration, that is, isolated voice (a), two (b), three (c), and four voices (d). For five and higher prime number of voices, the most compact unity (i.e., the two-voice block in the case of five) shall be placed in the superior portion of the voicing. In case of cardinalities with two possible segmentations (6 = 2 + 2 + 2 = 3 + 3, and 8 = 2 + 2 + 2 + 2 = 3 + 3 + 2), choose the option with lesser number of blocks (i.e., the latter options in the two cases).

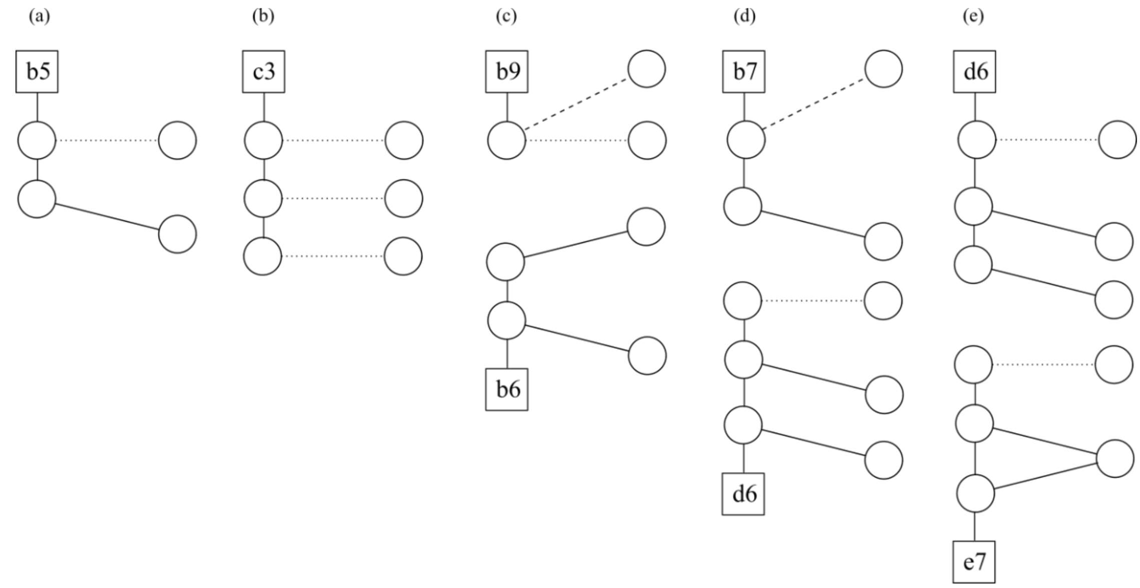


Figure 16: Examples of segmentation of voices into PVLs according to the number of linkages involved: (a) two; (b) three; (c) four; (d) five; (e) six.

5. Analyses

This section examines under the theoretical-methodological framework above described some examples from the harmonic universe covered by the study. As mentioned in the introductory notes, Jobim's music can be considered as a perfect representative of this context. Short excerpts of five of his most known songs were selected. It is noteworthy to inform that all voicings used the examples are harmonic reductions based on strict transcriptions of Jobim's own performances at the piano.[32] The adoption of Jobinian original voicings for basing the analysis prevents these from being considered biased in anyway, as it would be suggested if voicing arrangements were built from only a sequence of chord labels.

[32] These transcriptions (that are in most cases ratified by the Jobim's autographs) are distributed in five volumes of the *Cancioneiro Jobim* (2006), sponsored by the Tom Jobim Institute, based in Rio. The individual scores are also available in pdf format in the website of the institute.

5.1 Corcovado

For an initial application, let us examine the first phrase of "Corcovado".[33] Figure 17 introduces the model for this and the further analyses. It includes: (a) harmonic reduction related to the piano voicings, in musical notation with corresponding chord labels; (b) voice-linking analysis, plotted in the format of a pitch-column graph. Linkages are established according to the preference rules (see subsection 2.3), serving as basis for the construction of (c) the c-PVL network.

Observe how the texture of five parsimonious voices (segmented into 2+3 blocks) is almost entirely maintained along the passage. Only the fourth binary relation it is reduced to a pair of two-voice blocks, but a divergent connection brings then back a fifth voice. Another relevant aspect is the presence of a chromatic-like descending trajectory in the bass, a landmark of Jobim's music (recall Freeman's words in the introduction). In contrast, the remaining voices behavior in a more "erratic" manner, which gives to the excerpt a rich picture in terms of relative movement between voices: all five types (identity, parallel, similar, oblique, and contrary) are present. Observe also how the pairs "low/treble" movements differ at each binary relation, with the exception of the last one, a double contrary motion (i.e., inside each block), with is somewhat consistent with the idea of harmonic closure at this point of the phrase.

[33] The original score of "Corcovado" ("Quiet Nights" in the English title) can be accessed at the link:

http://www.jobim.org/jobim/bitstream/handle/2010/10402/corcovado.pdf?sequence=1.

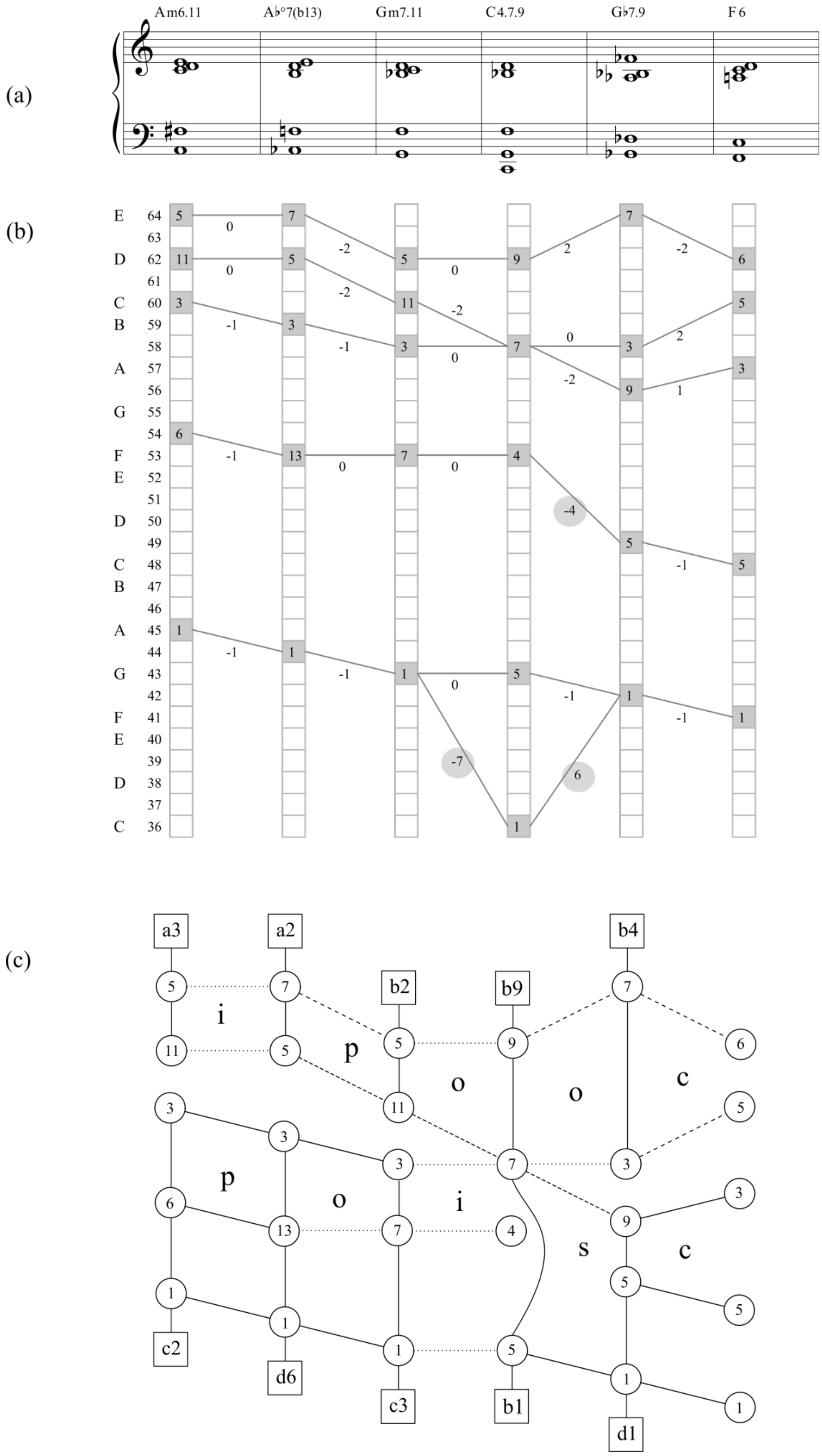


Figure 17: Analysis of "Corcovado" (mm.1–6): (a) harmonic reduction; (b) voice-linking (non-parsimonious intervals are circled); (c) c-PVL network (letters i, p, s, o, and c indicate relative movements between voices).

5.2 Samba de Uma Nota Só / Inútil Paisagem

Harmonic connections with pure c-PVLs (i.e., formed exclusively by parsimonious movements between voices) are not rare in Jobim's music. Figure 18 and Figure 19 depict the opening phrases of two well-known songs: “Samba de Uma Nota Só” (Jobim & Newton Mendonça) and “Inútil Paisagem” (Jobim & Aloysio de Oliveira).[34] These passages are based on two similar compositional strategies, considering the relationship between their respective reduced melodies (represented by the topmost voices in the examples) and bass lines, which once again descend chromatically in both cases. Whereas in “Samba de Uma Nota Só” the melody is intentionally maintained perfectly static (picturing the "unique note" D mentioned in the title and lyrics),[35] the melodic line of “Inútil Paisagem” mirrors the bass, describing a chromatic ascending trajectory.[36] The characteristic harmonic parallelism of both passages is evidenced not only by maintenance of functional labels of the notes, but also by the prominence of type p (parallel) of relative movements inside the individual bocks. Observe especially how sevenths follow other sevenths, in opposition of what occurs in normative situations, when seventh chords are chained by root movement of fifth.[37]

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[34] The pdf score of “Samba de Uma Nota Só” (“One-Note Samba” in the English title) can be downloaded at the link:

http://www.jobim.org/jobim/bitstream/handle/2010/4926/samba%20de%20uma%20nota%20so.pdf?sequence=2. For “Inútil Paisagem” (“If You Never Come To Me”) access:

http://www.jobim.org/jobim/bitstream/handle/2010/4824/inutil%20paisagem.pdf?sequence=2.

[35] For a detailed study about the remarkable implications of the lyrics of this song on several aspects of its music structure, see Almada (2009).

[36] This sort of contrapuntal opposition of chromatic voices is described by Robert Gauldin 2004 as a *chromatic wedge progression*. It can be directly connected with Leonard Meyer’s (1989) concept of *divergent wedge*, associated with a commonly used constructive procedure by Romantic composers, as well as with the basic notion of *omnibus progression* (Yellin 1998; Rockwell 2009).

[37] Which provokes guide-tone lines alternating sevenths and thirds, as exemplified in Figure 13a. As mentioned in section 1, Smither 2019 addresses systematically this specific line adopting a transformational approach.

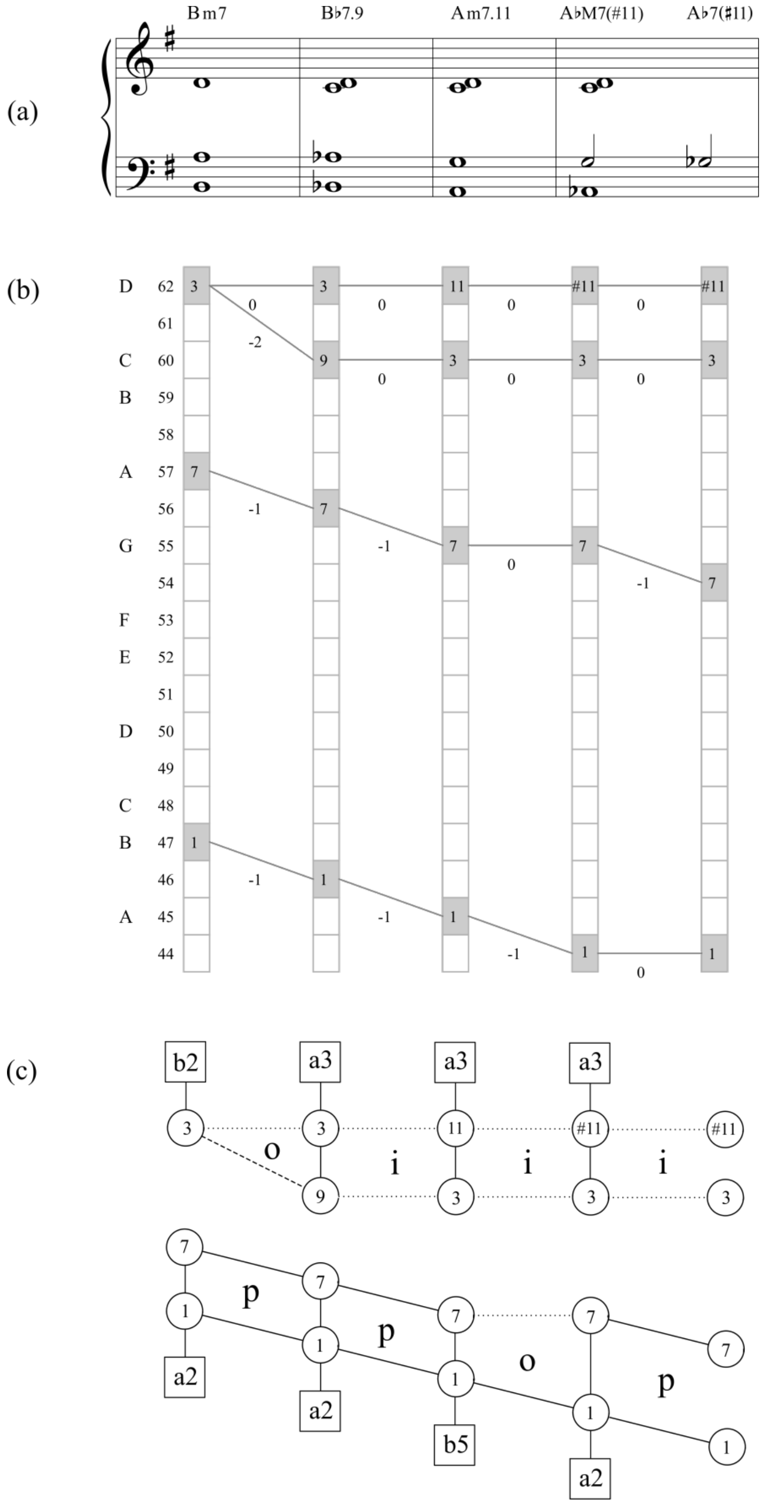

Figure 18: "Samba de Uma Nota Só" (mm. 1–4): (a) harmonic reduction; (b) voice-linking; (c) c-PVL network.

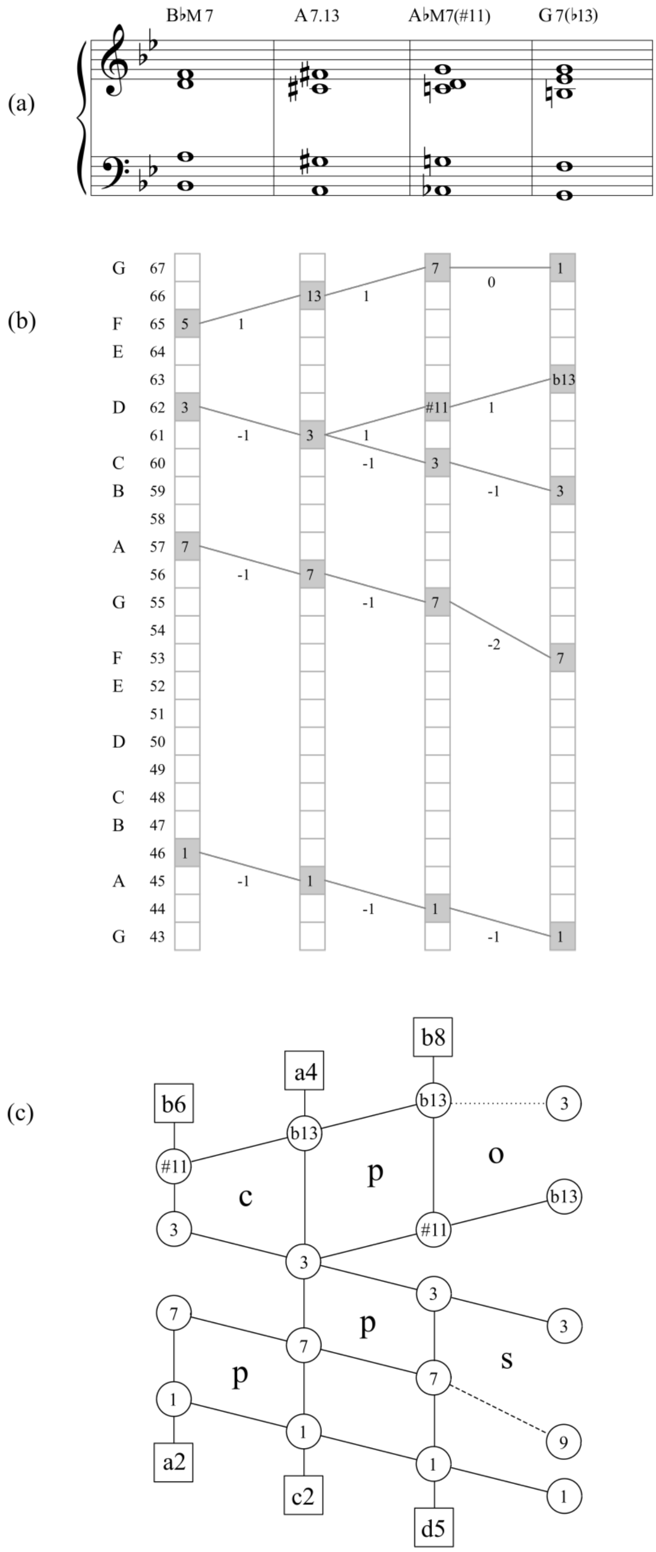

Figure 19: "Inútil Paisagem" (mm. 1-4): (a) harmonic reduction; (b) voice-linking; (c) c-PVL network.

From the examination of these two short, but very peculiar passages in respect of the behavior of their component voices arises a new possibility for extending the analysis beyond, by considering also PVL reductions.[38] Under this new perspective, quite similarly to conventional Schenkerian analysis, high-level and more structural voice-leading representations can be obtained by eliminating or collapsing/grouping non-essential melodic connections from a given "middleground" context.[39] Figure 20.a-b proposes an interpretation for the "Samba" as an underlying long-range oblique motion (class member b5) between the static top voice and the chromatic bass.[40] A more complex situation takes place in "Inútil Passagem": as shown in (c), the divergent wedge involving the melody and the bass line is replicated from the second chord on in two internal voices, as a sort of canon. This fractal-like organization is perfectly captured in the high-level PVL graph (d).

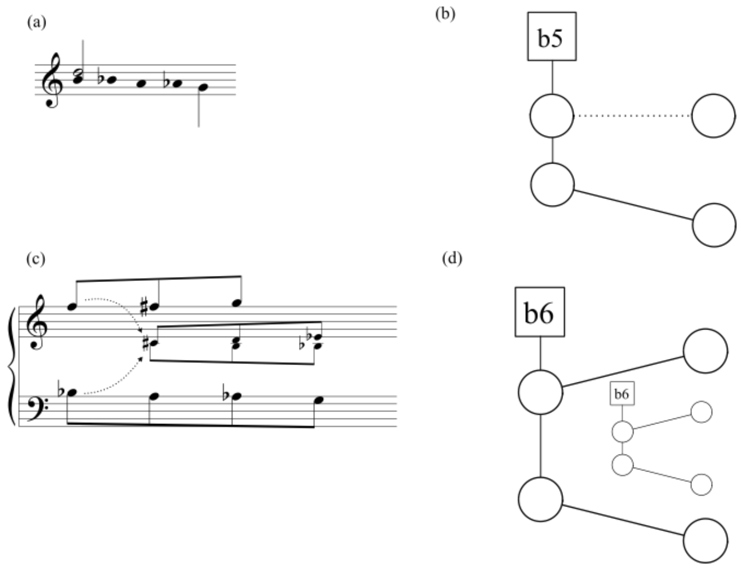

Figure 20: High-level PVL analysis of "Samba de Uma Nota Só" (a-b) and "Inútil Paisagem" (c-d).

[38] Since it involves deeper and still more abstract prospection of voicing paths, the new analysis addresses PVL configurations, rather than superficial c-PVLs.

[39] In this sense, eventual adjustments of register can be adopted aiming at more concise and simpler visualization.

[40] As suggested in the Schenkerian graph (a), the interaction of melody and bass can be seen as an unfolding of a G major triad, I of the key. Interestingly, this chord is not physically present in the progression (see harmony of Figure 18). Its first occurrence takes place only at the closure of first part (m.16).

5.3 Eu Te Amo / Chovendo na Roseira

The next two examples bring to discussion more lengthy chromatic descending lines.[41] In the first case (Figure 21), corresponding to the initial phrase of "Eu Te Amo" (Jobim & Chico Buarque de Hollanda),[42] the long minor-sixth descent performed almost entirely in parallel tenths by the bottom and top voices has clearly an expressive motivation, connoting despair, disorientation, and deep sadness (the lyrics describe an intense suffering of a man who has just been abandoned by his lover). The distribution of voices is mostly organized as a pair of two-voice blocks, intercalated with three occurrences of (0,1) PVLs. The entrance of the third chord (BM7(#5)) coincides with a sudden increase of cardinality (three to five linkages), due to two divergent movements in the "soprano" and "tenor". The global sorrowful descending of chordal roots and thirds is adorned by contrary and oblique motions of the other voices in local moments, bringing some asymmetry (and interest) to the whole, diluting the overall impression of pure parallelism. This is reflected by the distribution of the five relative types: i (two occurrences), p (also two), s (four), o (three), and c (four).

In the third section of "Chovendo na Roseira" (Figure 22)[43] the chromatic descent develops internally, flanked by completely static top and bottom voices, permuting the arrangement of fixed and moving voices from "Samba de Uma Nota Só". While this basic organization is not so clear in the pitch-column graph (b), the c-PVL network evidences the chromatic line as a unique stream (see the suggested "bridge" indicated by the sign * connecting the two portions of the line). In contrast with "Eu Te Amo", PVLs are not so diversified in terms the relative-movement types. Parallel and oblique movements dominate the texture.

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[41] Contrary to the previous examples, the melodic lines of the next ones do not integrate the voicings.

[42] For the score, access:

http://www.jobim.org/jobim/bitstream/handle/2010/4806/eu%20te%20amo.pdf?sequence=2.

[43] Entitled as "Double Rainbow" in the English version. For the score, access:

http://www.jobim.org/jobim/bitstream/handle/2010/4770/chovendo%20na%20roseira.pdf?sequence=2.

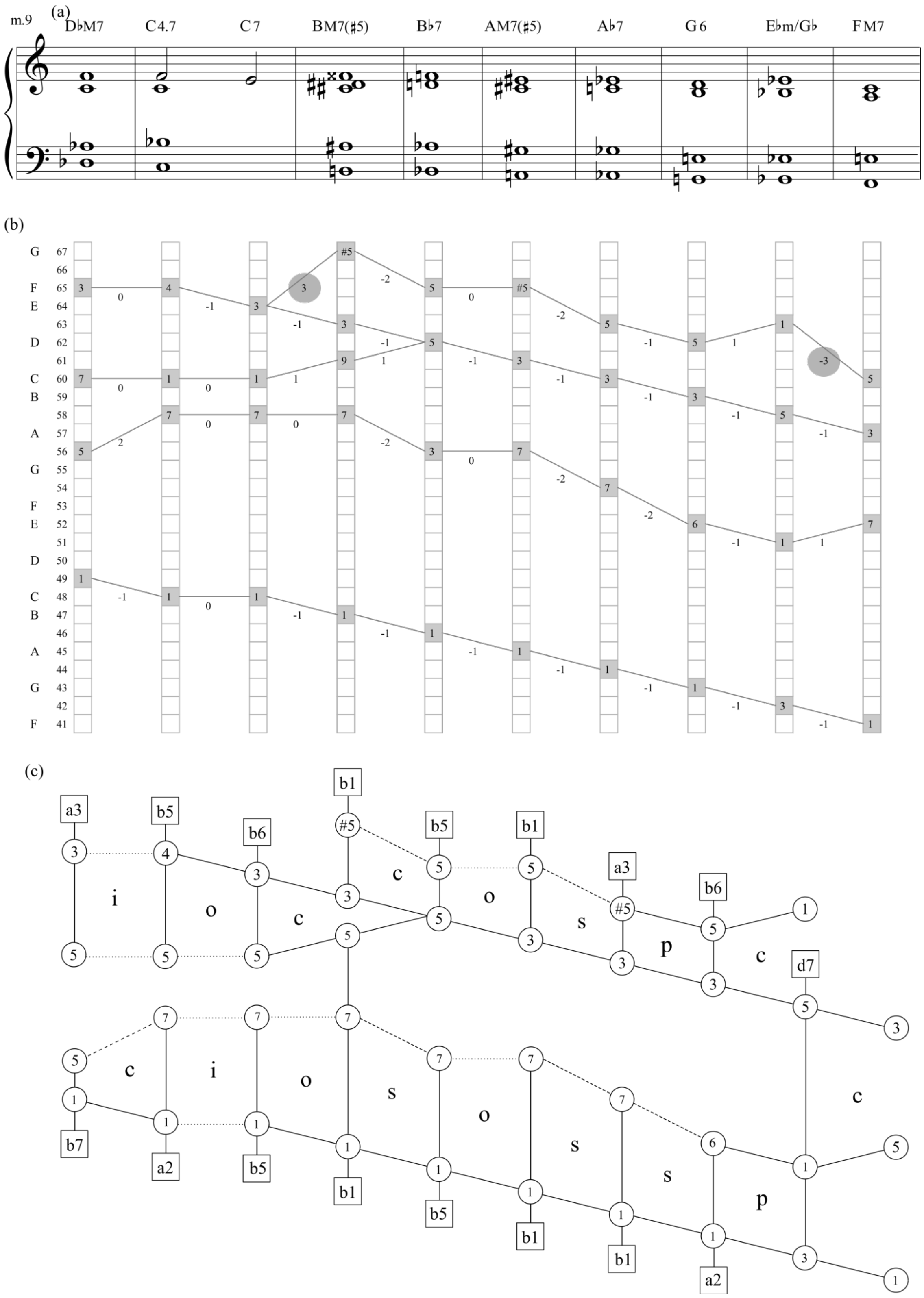

Figure 21: "Eu Te Amo" (mm. 9-17): (a) harmonic reduction; (b) voice-linking; (c) c-PVL network.

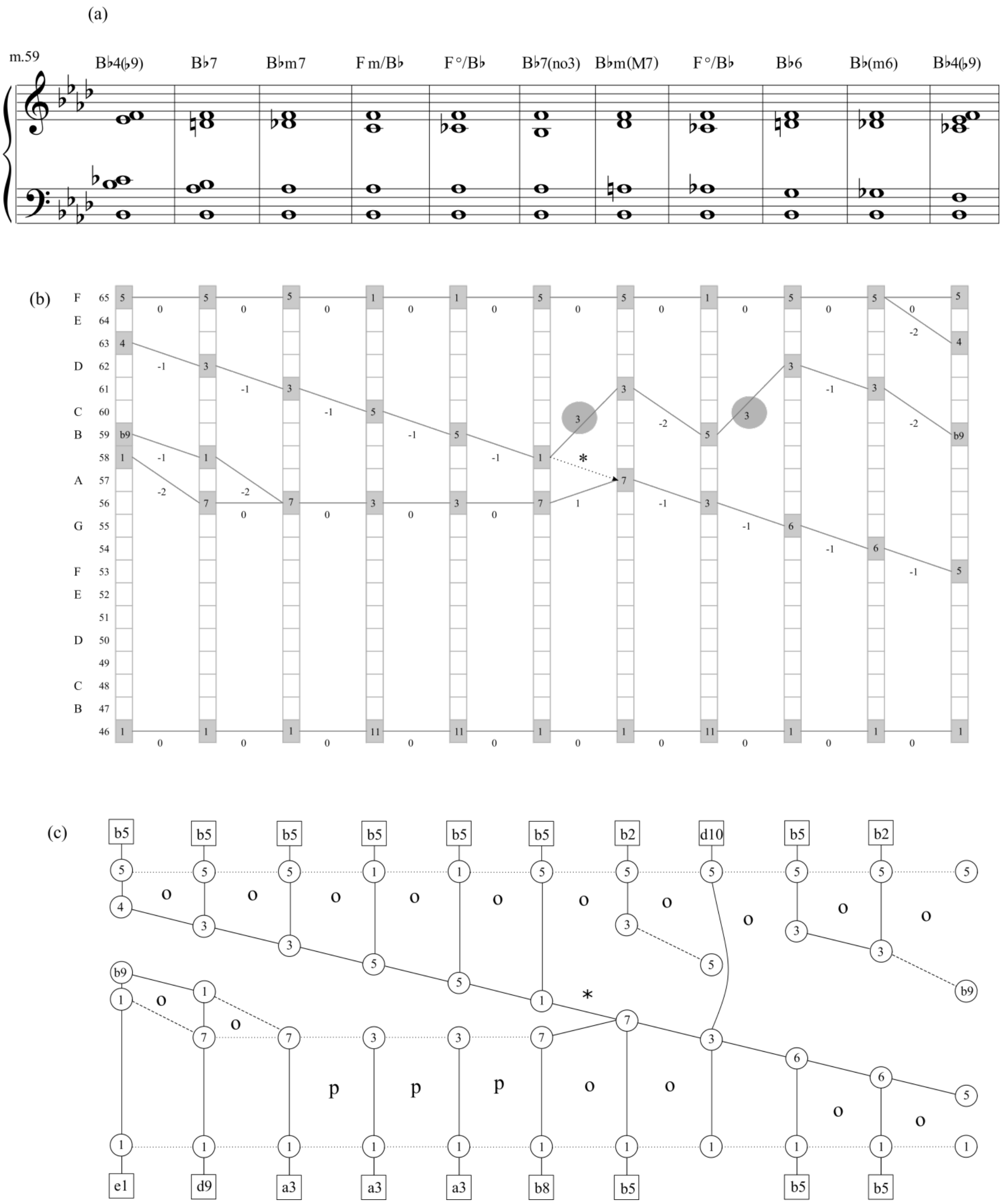

Figure 22: "Chovendo na Roseira" (mm. 59–69): voice-linking pitch graph (above) and c-PVL network (below).

This picture is reflected in deeper voice-leading organization. Figure 23.a depicts a Schenkerian-like interpretation of the passage, including the two pitches (F≅ and E≅) that lack for the completion of descending chromatic octave (both were omitted in Figure 22 for a question of space). Considering PVL's realm, a first reduction (b) defines high-level block d10 as possible descriptor for the passage. A further reduction (c) makes both stationary voices to merge into a unique stream, resulting into a basic b5 PVL, an essential synthesis of the passage (not casually, b5 is the most recurrent two-voice block in the excerpt).

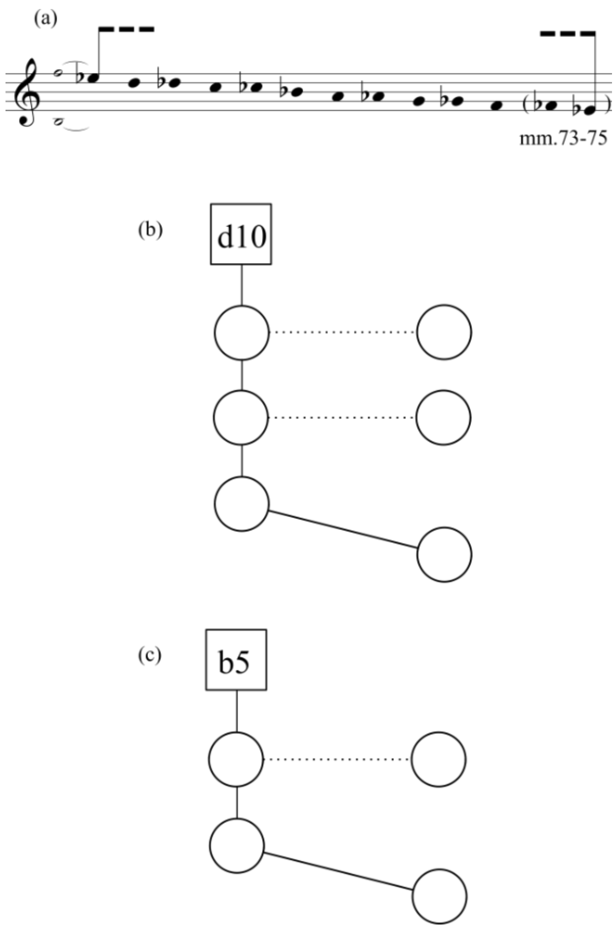

Figure 23: High-level PVL analysis of "Chovendo na Roseira".

5.4 Discussion

Although five short excerpts correspond obviously to a much reduced sample (and, therefore, inadequate in strict terms) for providing a comprehensive and consistent portrait of what we could call the "Jobinian voice leading", some information they convey points out to characteristic procedures, to be confirmed (or not) in further investigation, and probably generalized. Initially, let us consider the distribution of the intervallic linkage movements in the passages, informed by the pitch-column graphs (see

Table **5**).

		parsimonious					non-parsimonious
		– 2	– 1	0	+1	+2	
"Corcovado"	Σ	5	8	8	1	2	3
	%	18.5	29.6	29.6	3.7	7.4	11.1
"Samba de Uma Nota Só"	Σ	1	6	9	0	0	0
	%	6.3	37.5	56.2	0.0	0.0	0.0
"Inútil Paisagem"	Σ	1	8	1	4	0	0
	%	7.1	57.2	7.1	28.6	0.0	0.0
"Eu Te Amo"	Σ	5	18	7	4	1	2
	%	13.5	48.6	18.9	10.8	2.7	5.4
"Chovendo na Roseira"	Σ	5	11	24	1	0	2
	%	11.6	25.9	55.8	2.3	0.0	4.7
total	Σ	17	51	49	10	3	7
	%	12.4	**37.2**	35.8	7.3	2.2	5.1

Table 5: Comparison of the five analyses in respect to occurrence of parsimonious intervals.

Five especially meaningful considerations can be extracted from the examination of these numbers:

(a) Almost 95% of the connections are parsimonious;
(b) Minimal disturbance (0 and ±1 semitone) corresponds to 80.3% of the cases;
(c) Descending semitone is approximately five times more common than its ascending counterpart;[44]

[44] This is consistent with the phenomenon described by David Huron as *step declination*, namely the tendency of small melodic intervals to descend instead of ascending (Huron 2006, p. 75-77).

(d) A similar asymmetrical relation is observed between descending and ascending motions, considering together whole-tone and semitone steps: 49.6% vs. 9.5%;
(e) More than one third of the parsimonious linkages connect common tones.

Now concerning high-level organization,

Table **6** disposes the PVLs detected in the five analyses. Type b5 was largely the most used, with more than one fourth of the total. Taken together the member classes, oblique motion correspond to 41.1% of the cases.

	member	Γ	movement	recurrence	%
(1,0)	a2	<02000>	p	6	10.7
	a3	<00200>	i	9	16.1
	a4	<00020>	p	1	1.8
	b1	<11000>	s	6	10.7
	b2	<10100>	o	1	1.8
	b5	<01100>	o	**15**	**26.8**
	b6	<01010>	c	3	5.4
	b7	<01001>	c	1	1.8
	b8	<00110>	o	2	3.6
	b9	<00101>	o	1	1.8
(0,1)	c2	<03000>	p	2	3.6
	c3	<00300>	i	1	1.8
	d1	<21000>	s	1	1.8
	d5	<12000>	s	1	1.8
	d6	<02100>	o	1	1.8
	d7	<02010>	c	1	1.8
	d9	<10200>	o	1	1.8
	d10	<01200>	o	1	1.8
	e1	<11100>	o	1	1.8

Table 6: PVL member classes employed in the five analyses, considering labels, parsimonious vectors, type of relative movement, and recurrence (absolute numbers and percentages).

As a matter of fact, abstracted from the context of the Jobinian sample, b5 can be considered as the most optimal PVL, taking into account its inherent voice-leading efficiency, since it produces minimal dislocation (just one semitone) in the "right" direction, i.e., downwards.[45] Type b5 can be directly connected to seven other member classes (the two-voice a2, a3, b2, and b6, and the three-voice d6, d10, and e7), sharing with them the property of minimal disturbance, through two types of transformational "parsimonious" relations, as suggested in Figure 25: (a) “offset” (O), by changing of just one internal movement and keeping cardinality (relation denoted by the gray curved lines),[46] or (b) “addition” (A), denoted by straight black lines, by including a minimal-parsimonious linkage (0, +1 or -1), in this case transforming b5 into one of three related (0,1) blocks.[47] Observe also that other of (1,0) “derived” versions of b5 are A-related to the three-voice PVLs: it is the case of b6→e7, a2→e7 and a3→d10 (the indication of these relations were omitted only for the sake of visual clarity). This graph (let us call it a *PVL-lattice*) congregates the closest relations of a given PVL member class (called the *nucleus*). As a hypothesis to be explored, the PVLs that form this b5-lattice are largely preferred in Jobim’s practice (and, by extension, in other co-related contexts). This assumption can be initially supported by the findings of the present analysis. Back to Table 6, PVLs that integrate the lattice centered on b5 (shaded in background) represent, taken together, 64.4% of the occurrences.

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[45] Although still more economic (zero semitones) if compared to b5, the identity PVL a3 "fails" for not producing any melodic dislocation. Indeed, this class can be considered as an auxiliary block, having not an autonomous existence, but being frequently combined with other ones, as observed in the analyses.

[46] Superscripts indicate direction: clockwise (–) or counterclockwise (+).

[47] These operations can also be applied in reverse direction (for example, transforming b6 into b5).

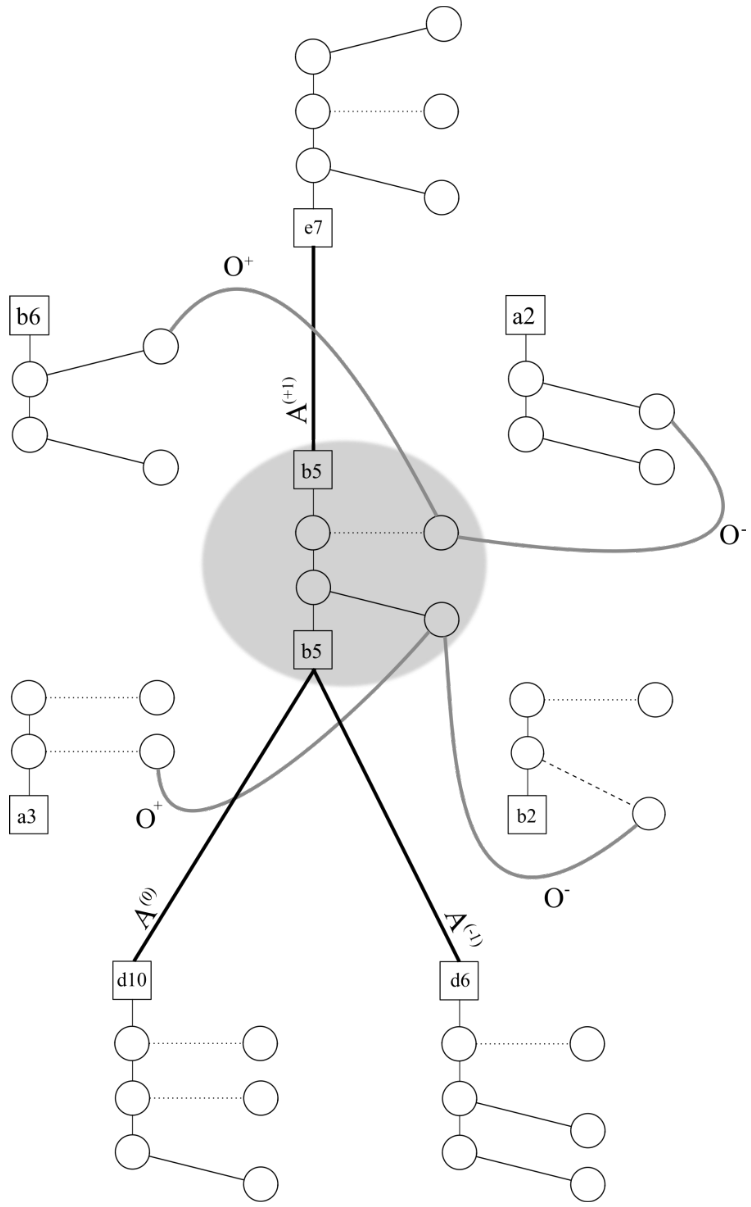

Figure 24: PVL- lattice referred to nucleus b5: gray lines denote operation "rotation" (R), and straight black lines "addition" (A).

6. Concluding remarks

This article introduced an original theory related to voice leading addressed to jazz, bossa, and co-related musical genres and subgenres. A number of concepts and typologies provided the necessary means for the formalization of a system of classes of parsimonious voice leading, or PVLs. Their contextual realization (c-PVLs) refines this theoretical framework and opens a number of practical applications, briefly envisaged with the examination of consecutive dominant formulas, and with the analysis of five short excerpts of Antônio Carlos (Tom) Jobim's songs.

Considering the latter application, even not being a sufficiently large sample to be taken as statistically representative of Jobim's harmonic modus operandi, the selected passages present some distinctive elements that were detected in the analysis: (1) descending chromatic lines (when in the bass, the rare usage of inversions imply chordal parallelism, one remarkable characteristic of Jobim's harmony); (2) parsimonious relations as rule in the melodic connections of the chordal voices, enhanced by use of harmonic extensions (eventually altered); (3) semitone oblique motion as ideal relation between melodic streams. In this respect, PVL b5, as above discussed, occupies probably a central position, as an essential building block for voice organization, since it not only promotes minimal-disturbance motion, but also favors "tectonic" (i.e., non-simultaneous) dislocation, resulting into very smooth and subtle melodic movement inside the harmonic progressions.

Certainly these characteristics are not exclusive of Jobim's palette, even though they probably integrate his music with unique intensity. In fact, such procedures are shared by other composers that inhabit the same broad aesthetical universe, which turns the theoretical-methodological framework here described suitable for further expansion and generalization. As a necessary step in this direction, future studies will properly extend the analytical investigation to different (and larger) corpora of pieces, searching to confirm or refute the present findings and conjectures. In this avenue to be explored, the use of the quantitative models applied in the last section of the article seems to be an adequate strategy.

Some new ideas open additional, attractive possibilities for related theoretical-analytical development. One of these is the concept of *voice-leading archetypes*, briefly introduced in Figure 13. Besides consecutive dominants, other recurrent harmonic formulas (like the characteristic "two-five" pattern) and their multitude of variants can be examined according to their potentialities for optimization of voice-leading efficiency. The modeling of these formulas is currently part of an ongoing project, revealing some very interesting c-PVL lineage-like patterns.

Another possible research path, as mentioned elsewhere, concerns a process of formalization based on group theory, focused on the symmetrical and transformational relations between PVLs. The systematical exploration of the parsimonious affinities between the classes (just initiated in the PVL-lattice graph of Figure 23) is certainly a very promising perspective for this theory.

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